

TAIL RISK EQUIVALENT LEVEL TRANSITION AND ITS APPLICATION FOR ESTIMATING EXTREME L_p -QUANTILES

Qingzhao Zhong and Yanxi Hou*

Fudan University

Abstract: L_p -quantiles, as a generalization of Value-at-Risk and expectile, have gained increasing attention in risk management due to their feasibility and straightforwardness in statistical implementation. This paper introduces the concept of Tail Risk Equivalent Level Transition (TRELt) to capture changes in tail risk when transitioning between two L_p -quantiles. Motivated by PELVE in Li and Wang (2003) but tailored for tail risk, we investigate the theoretical properties of TRELt, including its existence, uniqueness, and asymptotic behavior. Additionally, we develop inference methods for TRELt and extreme L_p -quantiles using this risk transition, which serves as a novel extrapolation technique in extreme value theory. Simulation studies and real data analysis demonstrate the empirical performance of these methods.

Key words and phrases: Extreme value theory, heavy-tailed data, L_p -quantile, TRELt.

1. Introduction

In recent years, a significant issue has emerged in the field of risk management: the transition between different risk measures. The central challenge is to develop a robust model that can accurately describe the shift from one risk measure to another. According to the Basel Committee on Banking Supervision (BCBS, 2019), banks are required to transition from Value-at-Risk (VaR) to Expected Shortfall (ES) as a more reliable metric for risk quantification. This recommendation is based on ES's coherence as a risk measure (Artzner et al., 1999) and its enhanced capacity to capture tail risk. The risk transition naturally leads to an important question: how do different risk measures transition and subsequently influence regulatory frameworks, particularly in terms of their relative conservatism?

To characterize the equivalent transition between VaR and ES while ensuring consistency in risk quantification, Li and Wang (2023) introduced the concept of Probability Equivalent Level of VaR and ES (PELVE), which identifies the balancing point for this transition. For a random loss X and a (tail) risk level

*Corresponding author. E-mail: yxhou@fudan.edu.cn.