

GENERALIZED TENSOR REGRESSION WITH INTERNAL VARIATION REGULARIZATION

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Abstract: Analyzing clinical diagnosis along with high-dimensional imaging data, while accounting for the piecewise constant nature of the imaging, presents challenges to existing statistical approaches. In this paper, we propose a generalized tensor regression framework with Internal Variation (IV) regularization to address these challenges. The inclusion of IV regularization allows for the explicit utilization of the rich spatial structure, particularly the piecewise constant nature of high-order imaging data, albeit with a more complex algorithm and demanding theoretical investigation. We develop an efficient IV regularized optimization procedure for estimating unknown scalar and tensor coefficients. We investigate the theoretical properties of scalar and tensor coefficient estimates, especially the error bounds of regularized tensor coefficient estimates. Extensive numerical studies assess the finite sample performance of our method, demonstrating its superiority over existing approaches. Finally, we apply the proposed method to a chronic sinusitis computed tomography (CT) imaging dataset and identify the most activated subregion across the maxillary sinus cavity associated with the diagnosis.

Key words and phrases: CP decomposition, internal variation, piecewise constant, regularized GLM, tensor regression.

1. Introduction

Tensor data, represented as multi-dimensional arrays, naturally arise in various fields such as imaging, neuroscience, and spatiotemporal analysis, where the data exhibit inherent structural dependencies. Tensor regression extends traditional regression models by leveraging these multi-way structures to capture complex relationships between tensor predictors and scalar or tensor outcomes, with widespread applications across various fields (Bi et al., 2021; Liu et al., 2022). In image processing, tensor regression has been used for tasks such as denoising (Zhang, Fu and Li, 2021), medical diagnosis (Zhou, Li and Zhu, 2013; Li and Zhang, 2021; Feng, Bi and Zhang, 2021), and brain connectivity analysis (Spencer, Guhaniyogi and Prado, 2019). In addition, it plays a crucial role in examining the relationship between manufacturing parameters and the geometry of manufactured parts (Yan, Paynabar and Pacella, 2019). In multi-task learning, tensor regression uses shared information across tasks, enhancing

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