

CENTER-OUTWARD RANKS AND SIGNS FOR TESTING CONDITIONAL QUANTILE INDEPENDENCE

Kai Xu, Huijun Shi and Daojiang He*

Anhui Normal University

Abstract: This article treats the problem of constructing nonparametric asymptotically distribution-free tests for conditional quantile independence in multidimensions. Our approach combines the quantile martingale difference divergence with a notion of the recently introduced multivariate center-outward ranks and signs. We derive the asymptotic null representation of the proposed test statistics by exploiting the degenerate V -type and U -type structures of the quantile martingale difference divergence and the Glivenko–Cantelli strong consistency and distribution-freeness of the center-outward ranks and signs. This representation permits direct calculation of limiting null distributions without requiring bootstrap calibration. We further show that our center-outward versions of the quantile martingale difference divergence tests are consistent against all fixed alternatives. A local power analysis provides strong support for the center-outward approach by establishing the nontrivial power of our center-outward rank-based tests over root- n neighborhoods. Moreover, the proposed tests are computationally feasible and well-defined without any moment assumptions. We illustrate the advantages of the proposed methods via extensive simulation studies and a gene expression dataset analysis.

Key words and phrases: Center-outward ranks and signs, conditional quantile, Hájek representation, Le Cam’s third lemma, quantile martingale difference divergence, V -type and U -type processes.

1. Introduction

Inference for regression models is one of the most important issues in statistics. Since Koenker and Bassett (1978), quantile regression has attracted increasing attention, mainly due to the robustness against outliers in the response and the ability to capture heterogeneity in the set of important predictors. More references about quantile regression estimation and interpretation can be found in the seminal book by Koenker (2005). For a scalar response variable $Y \in \mathbb{R}$ and a set of predictors $\mathbf{x} \in \mathbb{R}^q$, we are interested in exploring whether $\mathbf{x} = (X_1, \dots, X_q)^\top$ is useful in modeling a certain aspect of the quantiles of Y , before constructing a parametric/nonparametric quantile regression model. For theoretical development, a fixed dimension is assumed to derive the asymptotic results.

*Corresponding author. E-mail: djhe@anhu.edu.cn