

CATONI-TYPE CONFIDENCE SEQUENCES UNDER INFINITE VARIANCE

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Abstract: In this paper, we provide an extension of confidence sequences for settings where the variance of the data-generating distribution does not exist or is infinite. Confidence sequences furnish confidence intervals that are valid at arbitrary data-dependent stopping times, naturally having a wide range of applications. We first derive the Catoni-style confidence sequences for data distributions having a bounded p^{th} moment, where $p \in (1, 2)$, using Ville's inequality, and strengthen the existing upper bound results. The derived results are shown to be better than confidence sequences obtained using vanilla Dubins–Savage inequality. We next establish a lower bound for the width of the Catoni-style confidence sequences for $p \in (1, 2]$, and establish the statistical limitation of applying Ville's inequality based techniques to Catoni-style confidence sequence estimation. To close this gap, we further establish the tighter confidence sequences using the stitching methods. Our new methodology can be easily applied to risk control and parameter estimation problems.

Key words and phrases: Catoni estimator, confidence sequence, heavy tail, law of iterated logarithm.

1. Introduction

Sequential design of experiments is a classical framework in statistical sampling theory, in which the size and the composition of samples are not fixed in advance and are allowed to be functions of the observations themselves (Robbins, 1952). Confidence sequence (CS) is one particular tool in sequential design that facilitates anytime-valid inference (Darling and Robbins, 1967; Jamieson and Jain, 2018; Howard et al., 2021). In particular, a confidence sequence is a sequence of confidence intervals that is valid at data-dependent stopping times.

Formally, let $X_1, X_2, \dots$ be an independent and identically distributed (i.i.d.) stream drawn from distribution P . The basic object of interest is the unknown mean of this distribution, namely, $\mu = \mathbf{E}_P[X_1]$. A crude way to quantify the uncertainty associated with the mean estimation problem is via *confidence intervals*. Here one constructs a $\sigma(X_1, \dots, X_t)$ -measurable interval $\mathbf{CI}_t$ for each $t \in \mathbb{N}^+$ such that $\forall t \in \mathbb{N}^+$, the following holds: $\mathbb{P}(\mu \in \mathbf{CI}_t) \geq 1 - \alpha$,

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