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INFERENCE FOR DELAY DIFFERENTIAL EQUATIONS USING MANIFOLD-CONSTRAINED GAUSSIAN PROCESSES

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Abstract: Dynamic systems described by differential equations often involve feedback among system components. When there are time delays for components to sense and respond to feedback, delay differential equation (DDE) models are commonly used. This paper considers the problem of inferring unknown system parameters, including the time delays, from noisy and sparse experimental data observed from the system. We propose an extension of manifold-constrained Gaussian processes to conduct parameter inference for DDEs, whereas the time delay parameters have posed a challenge for existing methods that bypass numerical solvers. Our method uses a Bayesian framework to impose a Gaussian process model over the system trajectory, conditioned on the manifold constraint that satisfies the DDEs. For efficient computation, a linear interpolation scheme is developed to approximate the values of the time-delayed system outputs, along with corresponding theoretical error bounds on the approximated derivatives. Two simulation examples, based on Hutchinson's equation and the *lac* operon system, together with a real-world application using Ontario COVID-19 data, are used to illustrate the efficacy of our method.

Key words and phrases: Bayesian inference, dynamic systems, Hutchinson's equation, *lac* operon, parameter estimation.

1. Introduction

Delay differential equations (DDEs) are commonly used to model dynamic processes with time delays in ecology (Hutchinson, 1948), biology (Yildirim and Mackey, 2003), and epidemiology (Ma, Song and Takeuchi, 2004). In this setting, components of the system require time to respond to feedback, thereby introducing time delay parameters into the model. For example, when modeling disease transmission mechanisms, the system of DDEs could incorporate constant time delays to accommodate the incubation period of the infectious disease (Ma, Song and Takeuchi, 2004). In models for gene regulation, the time required for transcription and translation activities could be encoded via time delay parameters (Yildirim and Mackey, 2003).

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