

SUFFICIENT DIMENSION REDUCTION FOR THE CONDITIONAL QUANTILES OF FUNCTIONAL DATA

Eliana Christou, Eftychia Solea, Shanshan Wang and Jun Song*

*University of North Carolina at Charlotte, Queen Mary,
University of London and Korea University*

Abstract: Functional data analysis holds transformative potential across fields but often relies on mean regression, with limited focus on quantile regression. Furthermore, the infinite-dimensional nature of the functional predictors necessitates the use of dimension reduction techniques. Therefore, in this work, we address this gap by developing dimension reduction techniques for the conditional quantiles of functional data. The idea is to replace the functional predictors with a few finite predictors without losing important information on the conditional quantile while maintaining a flexible nonparametric model. We derive the convergence rates of the proposed estimators and demonstrate their finite sample performance using simulations and a real dataset from fMRI studies.

Key words and phrases: Conditional quantiles, dimension reduction, functional data analysis.

1. Introduction

Functional data analysis (FDA) considers data as realizations of random functions, impacting fields like finance (Müller, Sen and Stadtmüller, 2011), natural language processing (NLP) (Gubian, Torreira and Boves, 2015), electric grid stabilization (Elías, Jiménez and Shang, 2022; Fontana, Tavoni and Vantini, 2019), and notably medicine (Li and Luo, 2017; Pratt et al., 2021). For example, neuroimaging studies on disorders like attention deficit hyperactivity disorder (ADHD) or Alzheimer’s disease aim to understand neural development, substance use, and brain structure and function. These studies often collect functional data such as functional magnetic resonance imaging (fMRI) and electroencephalogram (EEG). While traditionally analyzed using conventional methods, FDA offers a more precise representation along the continuum.

FDA, first coined by Ramsay and Dalzell (1991), has been a very active research area. Key works on functional linear regression include Cardot, Ferraty and Sarda (2003), Yao, Müller and Wang (2005), Cai and Hall (2006), Hall and Horowitz (2007), Crambes, Kneip and Sarda (2009), Shin (2009), James, Wang and Zhu (2009), and Yuan and Cai (2010); see also the monographs of Ramsay and Silverman (2005) and Kokoszka and Reimherr (2017). Nonparametric

*Corresponding author. E-mail: junsong@korea.ac.kr