Statistica Sinica **37** (2027), 1-23
doi:https://doi.org/10.5705/ss.202024.0196

DISTINGUISHING FORMS OF ASYMPTOTIC DEPENDENCE IN HEAVY TAILED DATA

Tiandong Wang*[1,2] and Sidney I. Resnick[3]

[1] *Fudan University,* [2] *Shanghai Academy of Artificial Intelligence for Science and* [3] *Cornell University*

Abstract: In multivariate heavy tail estimation, the support of the limit measure provides information on the asymptotic dependence structure of the random vector with the heavy tail distribution. This asymptotic dependence structure may be difficult to discern, even in favorable cases of $\mathbb{R}_+^2$-valued data since exploratory methods can be ambiguous and heavily dependent on threshold choice. We restrict ourselves to techniques that help distinguish between the following asymptotic models for heavy tails on $\mathbb{R}_+^2$: (i) full dependence where the limit measure concentrates on a ray from the origin; (ii) strong dependence where the support of the limit measure is a proper connected subcone of the positive quadrant; (iii) weak dependence where the limit measure concentrates on the whole positive quadrant. We propose two test statistics, analyze their asymptotically normal behavior under full and not-full dependence, and discuss method implementation using bootstrap methods. The methodology is illustrated with both simulated and real data.

Key words and phrases: Asymptotic dependence, hidden regular variation, multivariate extremes.

1. Introduction

In multivariate heavy tail estimation, the support of the limit measure provides information on the dependence structure of the random vector with the heavy tail distribution (Lehtomaa and Resnick, 2020). However, even in simple circumstances in $\mathbb{R}_+^2$, the positive quadrant in two dimensions, exploratory methods such as scatter, diamond or density estimation plots may have trouble distinguishing between cases:

- *Full dependence* where the limit measure concentrates on a ray of the form $\{(x, y) \in \mathbb{R}_+^2 : y/x = m > 0\}$;
- *Strong dependence* (See Fig. 2) where the support of the limit measure is a proper *connected* subcone of $\mathbb{R}_+^2$ of the form $\{(x, y) \in \mathbb{R}_+^2 : y/x \in [m_l, m_u] \subsetneq [0, 1]\}$;

*Corresponding author. E-mail: td_wang@fudan.edu.cn