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CONVOLUTED SUPPORT MATRIX MACHINE IN HIGH DIMENSIONS

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Abstract: The Support Vector Machine (SVM) has been effective in various discrimination problems. Recently, there has been growing interest in extending the traditional vector-based SVM to accommodate structured matrix inputs. However, the nonsmooth hinge loss poses significant challenges for both theoretical and computational development. To address these issues, we propose a convex smoothing procedure for the hinge loss. Additionally, we introduce an elastic-net type penalty to handle high-dimensional matrix inputs. Our approach surpasses the standard SVM for discrimination involving high-dimensional matrix inputs. The proposed method provably achieves an optimal statistical convergence rate, and the smooth, convex loss function enables the development of a highly efficient optimization algorithm. This algorithm features a fast linear convergence rate and a simple implementation. Extensive simulations and an electroencephalography application demonstrate the method's superiority in classification accuracy and computational efficiency.

Key words and phrases: Asymptotic theory, convolution-type smoothing, high-dimensional matrix regression, linear support vector machines.

1. Introduction

Since its inception by Boser, Guyon and Vapnik (1992) and Vapnik (2000), the Support Vector Machine (SVM) has become a key tool for discrimination problems in a broad range of applications, including pattern recognition, computer vision, and disease diagnosis (Bishop, 2006). The statistical properties of SVM have been well-explored (Steinwart and Scovel, 2007; Eberts and Steinwart, 2013), with recent work by Cui et al. (2022) providing explicit error rates for nonlinear SVM.

Despite its success, applying standard SVM to real-world problems presents significant challenges. The first challenge is that standard SVM is primarily designed for vector inputs, while many practical discrimination tasks involve more structured inputs, such as matrices. In applications like computer vision or medical diagnosis, images are typically represented as matrices where each pixel corresponds to a matrix entry. Reshaping these matrices into vectors can disrupt the inherent structural information and produce very high-dimensional vectors,

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