Statistica Sinica **37** (2027), 1-29
doi:https://doi.org/10.5705/ss.202024.0191

CHARACTERIZING AND COMPARING ORDER-OF-ADDITION ORTHOGONAL ARRAYS

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Abstract: Given a set of parameters, several non-isomorphic order-of-addition orthogonal arrays can be generated to design an order-of-addition experiment. Under resource constraints, selecting the best from these candidate designs for the experiment can be practical to extract as much information as possible from the observed data. Based on some theoretical results developed for two-level orthogonal arrays, a series of numerical indices called centralized generalized wordlength pattern is proposed in this paper to characterize and compare order-of-addition orthogonal arrays. Specifically, the J-characteristics are first justified for pairwise order matrices when the transitive property of pairwise order factors is taken into account. The centralized generalized wordlength pattern is then defined based on the sums of squared differences between the normalized J-characteristics of the pairwise order matrices determined by the fractional and full designs. Essentially, it can be viewed as a natural extension of the generalized wordlength pattern used for two-level orthogonal arrays. Their functional relationship is further simplified such that the computational cost can be reduced significantly. Some optimal order-of-addition orthogonal arrays with economical run sizes are identified from existing catalogues for future work.

Key words and phrases: Hadamard matrix, Hamming distance, inversion, J-characteristic, projection property.

1. Introduction

Order-of-addition experiments are often conducted to explore optimal addition orders of several components in agricultural, chemical, industrial and pharmaceutical studies. Some real-world order-of-addition problems were introduced by Voelkel and Gallagher (2019) and Wang, Xu and Ding (2020). Because every component is fixed at a constant level throughout an experiment, the concept of factors used in conventional theory of experimental designs cannot be applied directly when designing an order-of-addition experiment. Therefore, efficient designs for such experiments have received increasing attention from researchers and practitioners in recent years. Van Nostrand (1995) first used a series of pseudo factors taking values ± 1 to denote whether or not a particular component is added before another component. Following this line of thought, Voelkel (2019) proposed order-of-addition orthogonal arrays to design these kinds

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