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ON EFFICIENT ESTIMATION FOR VALUE-AT-RISK VIA LOCATION-SCALE TIME SERIES MODELS

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Abstract: This paper proposes efficient estimation methods for Value-at-Risk (VaR) in the framework of location-scale time series models, including the semi-parametric and parametric composite quantile regression (CQR). The semi-parametric CQR does not impose any distribution assumptions on the innovations, while the parametric CQR assumes that the innovations follow some distributions with explicit and parametric quantile functions. Compared with the quantile regression, the semi-parametric CQR method improves estimation efficiency by combining data information at multiple quantile levels. The parametric CQR takes advantage of model flexibility, and can further enhance efficiency in face of data scarcity when estimating high conditional quantiles. We establish the asymptotic properties of both CQR methods for location-scale time series models, and particularly for the ARMA-GARCH, double autoregressive and NAR-GARCH type models. We also compare both CQR estimators in estimation efficiency, and compare them with the Gaussian and exponential quasi-maximum likelihood estimators. Finally, we examine the finite-sample performance of the proposed methods via simulation studies, and analyze an empirical dataset to illustrate their usefulness in modeling and forecasting VaR for financial assets.

Key words and phrases: ARMA-GARCH models, composite quantile regression, double autoregressive models, location-scale, value-at-risk.

1. Introduction

In risk management, value-at-risk (VaR) is a widely used indicator to measure market risk. It quantifies the maximum potential loss of an asset or portfolio over a specified period at a given confidence level. Various models have been proposed for VaR and they can be classified into three groups: non-parametric, parametric and semi-parametric methods. Non-parametric methods, such as historical simulation (HS) in Barone-Adessi, Giannopoulos and Vosper (1999) and the Boudoukh, Richardson, and Whitelaw (BRW) method in Boudoukh, Richardson and Whitelaw (1998), neither require the assumption of return distribution nor parameterize conditional volatility. As a result, they are easy to implement but may yield less accurate results when faced with highly fluctuating data. On the other hand, parametric methods, including Risk Metrics in Morgan and Reuters (1996), not only require the modeling and parameterization of

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