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DIMENSION REDUCTION FOR EXTREME REGRESSION VIA CONTOUR PROJECTION

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Abstract: In extreme regression problems, a primary objective is to infer extreme values of the response given a set of predictors. The high dimensionality and heavy-tailedness of the predictors limit the applicability of classical tools for inferring conditional extremes. In this paper, we focus on the central extreme subspace (CES), whose existence and uniqueness are guaranteed under fairly mild conditions. By projecting the data onto the CES, the dimension of the predictors is reduced while all the information for inferring conditional extremes is retained, which effectively addresses the high dimensionality issue. We propose the novel COPES method to estimate the CES by utilizing contour projection. Notably, COPES is robust against heavy-tailed predictors. The theoretical justification for the consistency of COPES is established. Overall, our proposal not only extends the toolkit for extreme regression but also broadens the scope of dimension reduction techniques. The effectiveness of our proposal is demonstrated through extensive simulation studies and an application to Chinese stock market data.

Keywords and phrases: Conditional extremes, elliptically contoured distribution, heavy-tailedness, sufficient dimension reduction.

1. Introduction

In regression problems, predicting the response becomes particularly challenging when the dimension of predictors gets large. This issue is more pronounced when we focus on the tail behaviour of the response, such as extreme quantile (Chernozhukov, 2005), extreme expectile (Girard, Stupfler and Usseglio-Carleve, 2021) and extreme probability (Hall and Weissman, 1997), which is referred to as *extreme regression*. The primary reason for this challenge is that when we study the tail of a distribution, the effective sample size is only a small fraction of the total sample size. Therefore, even moderate-dimensional predictors can easily deteriorate the state-of-the-art methods. To mitigate the impact of high dimensionality in extreme regression, some progress has been made in reducing the dimension of the predictors (Aghbalou et al., 2024; Gardes, 2018; Bousebata, Enjolras and Girard, 2023). Most existing methods for dimension reduction in extreme regression are grounded in the concept of sufficient dimension reduction (SDR) (Li, 1991; Cook, 1998), which seeks a

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