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BAYESIAN STATISTICS BY ARITHMETIC OPERATIONS OF CONJUGATE DISTRIBUTIONS

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Abstract: Conjugate distributions provide an entry point to Bayesian analysis. By defining summation, subtraction, and multiplication operators for conjugate distributions, we study Bayesian statistics by arithmetic operations. A striking feature is that the non-informative prior fulfills the central role of zero in mathematics. The summation operator connects Bayesian and frequentist estimators by a simple equation, which also provides an efficient method for evaluating the marginal likelihood. The subtraction operator facilitates cross-validation, rolling-window estimation, and regression under multicollinearity. The multiplication operator simplifies the weighted regression with a discount factor. Arithmetic operations conceptualize pseudo data in the conjugate prior, sufficient statistics that determine the likelihood, and the posterior that balances the prior and data.

Key words and phrases: Conjugacy, exponential family, linear regression, statistics education.

1. Introduction

On the one hand, summation, subtraction, and multiplication are basic arithmetic operators essential for primary education of mathematics. On the other hand, normal, beta and gamma priors are common conjugate distributions essential for introductory courses of Bayesian statistics. Traditionally, they are separate topics studied in different areas. We bridge the gap by defining arithmetic operators for conjugate distributions. Bayesian textbook materials, such as normal linear regressions, beta-binomial and gamma-Poisson models, are presented from a new perspective. The conjugate arithmetic operations provide novel pedagogical methods for studying

(1) the conjugate prior that incorporates pseudo data,

(2) the non-informative prior that accomplishes the role of zero in mathematics,

(3) the sufficient statistics that determine the likelihood function, and

(4) the posterior that balances data and the prior information.

Section 2 is the core of the paper, focusing on the theory and applications of normal-inverse-gamma (NIG) arithmetic for linear regressions. Section 3 extends

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