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AN AUTOMATIC MDDM-BASED TEST FOR MARTINGALE DIFFERENCE HYPOTHESIS

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Abstract: Checking whether the error term is a marginal difference sequence (MDS) in the multivariate time series model with a parametric conditional mean is a crucial problem. Tests based on the martingale difference divergence matrix (MDDM) are an effective statistical method for testing MDS in the residuals of multivariate time series models. However, MDDM-based tests require specifying the lag order. To solve this problem, we propose a data-driven MDDM-based test that automatically selects the lag order. This method has three main advantages: first, researchers do not need to specify the lag order while the test automatically selects it from the data; second, under the null hypothesis, the lag order is one; third, the proposed automatic tests have good performance in detecting model inadequacy caused by high-order dependence. In theory, we prove the asymptotical property of the proposed method. Furthermore, we demonstrate the effectiveness of this method through simulations and real data analysis.

Key words and phrases: Marginal difference sequence (MDS), martingale difference divergence matrix (MDDM), multivariate time series model.

1. Introduction

The concept of marginal difference sequence (MDS) is central in many areas of economics and finance. Many economic theories in a dynamic context, such as market efficiency hypothesis, rational expectations, or optimal consumption smoothing, deliver such dependence restrictions on the underlying economic variables, e.g., Hall (1978) and Lo and MacKinlay (1997). The martingale difference hypothesis (MDH) states that the best predictor, in the sense of least mean squared error, of the future values of a time series given the current information set is just the unconditional expectation. Hence, past information does not help to improve the forecast of future values of an MDS. Furthermore, MDH can be used to test the error term with parametric conditional mean for the time series model. More formally, people consider a time series

$$Y_t = m(\mathcal{I}_{t-1}) + \varepsilon_t, \tag{1.1}$$

where $Y_t \in \mathcal{R}^p$; $m(\mathcal{I}_{t-1}) = E(Y_t|\mathcal{I}_{t-1})$ is the conditional mean almost surely (a.s.) of Y_t given the conditioning set $\mathcal{I}_{t-1}$; $\varepsilon_t = Y_t - E(Y_t|\mathcal{I}_{t-1})$ by construction

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