Statistica Sinica **36** (2026), 1-21
doi:https://doi.org/10.5705/ss.202024.0028

FUNCTIONAL LINEAR MODELS WITH LATENT FACTORS

Zixuan Han[1], Tao Li[1], Jinhong You[1] and Jiguo Cao*[2]

[1]*Shanghai University of Finance and Economics and* [2]*Simon Fraser University*

Abstract: We propose a novel functional linear model incorporating latent factors, where scalar response, scalar covariates, and functional covariates have repeated measurements for each subject. Our model accounts for latent factors that may impact the response but remain unobservable. To unveil and estimate these latent factors, we propose an iterated profile estimation method. We then establish the consistency and asymptotic properties of the estimators. To demonstrate the efficacy of our proposed estimation procedure, we conduct simulation studies across various scenarios. We compare our results with estimations derived from conventional functional linear models, revealing the superior performance of our method in addressing latent factors. We further illustrate our proposed model and methodology by analyzing real data from both financial markets and air pollution datasets. In these analyses, we successfully uncover hidden factors that exert influence in these specific fields.

Key words and phrases: Factor model, functional data analysis, functional regression, penalized spline, profile estimation.

1. Introduction

Functional data refers to a type of data that consists of observations or measurements that are functions rather than simple numerical values or categorical variables (Ramsay, 1982). In other words, instead of representing data points as individual values or discrete categories, functional data captures information in the form of entire curves, trajectories, or continuous functions. Functional data analysis (FDA) is a statistical framework and set of techniques used to analyze and interpret functional data (Ramsay and Silverman, 2005; Ferraty and Vieu, 2006). It involves treating functions as the fundamental unit of analysis and applying statistical methods specifically designed for functional data. Functional regression is one of the most popular FDA tools (Chen et al., 2021; Liu, Ma and Chen, 2022). A great quantity of literature focused on this field can be divided into three categories based on whether the response variable is functional data (Jiang and Wang, 2011), the prediction variable is functional data (Hilgert, Mas and Verzelen, 2013), or both are functional data (Li, Huang and Zhu, 2017). Functional linear models can be used to model the relationship between functional variables and scalar response, which have been applied to many fields as financial

*Corresponding author. E-mail: jiguo_cao@sfu.ca