

TAIL GINI FUNCTIONAL UNDER ASYMPTOTIC INDEPENDENCE

Zhaowen Wang¹, Liujun Chen², and Deyuan Li^{*1}

¹*Fudan University* and ²*University of Science and Technology of China*

Abstract: Tail Gini functional is a measure of tail risk variability for systemic risks, and has many applications in banking, finance and insurance. Meanwhile, there is growing attention on asymptotic independent pairs in quantitative risk management. This paper addresses the estimation of the tail Gini functional under asymptotic independence. We first estimate the tail Gini functional at an intermediate level and then extrapolate it to the extreme tails. The asymptotic normalities of both the intermediate and extreme estimators are established. The simulation study shows that our estimator performs comparatively well in view of both bias and variance. The application to measure the tail variability of weekly loss of individual stocks given the occurrence of extreme events in the market index in Hong Kong Stock Exchange provides meaningful results, and leads to new insights in risk management.

Key words and phrases: Asymptotic independence, systemic risk, tail variability.

1. Introduction

Measuring tail risk is crucial in many different fields such as banking, finance and insurance. The most popular tail-based risk measures in the literature are Value at Risk (VaR) and Expected Shortfall (ES). See Patton, Ziegel and Chen (2019), Li and Wang (2023), and Hoga and Demetrescu (2023) for some recent discussions on VaR and ES in quantitative risk management. Due to their nature, VaR and ES do not capture the variability of the risk random variable beyond the high quantile. To incorporate variability in tail risk analysis, Furman, Wang and Zitikis (2017) extends the (classic) Gini mean difference to tail Gini functional. The tail Gini functional for X is defined by

$$\text{TG}_p(X) = \frac{4}{p} \text{Cov} \{X, F_1(X) \mid F_1(X) > 1 - p\},$$

where F_1 is the cumulative distribution function of random variable X and $p > 0$ is a sufficient small value. Obviously, the tail Gini functional $\text{TG}_p(X)$ is designed to measure the variability of X on the upper tail region and quantifies the risk of X solely. In finance, X may be the loss of an individual asset and tail Gini functional for X indicates the risk measure of tail variability of that asset.

^{*}Corresponding author. E-mail: deyuanli@fudan.edu.cn