Statistica Sinica **37** (2027), 1-30
doi:https://doi.org/10.5705/ss.202023.0415

FUNCTIONAL LINEAR OPERATOR QUANTILE REGRESSION FOR SPARSE LONGITUDINAL DATA

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Abstract: We propose a functional linear operator quantile regression (FLOQR) framework, which includes many important and useful functional data models, and devote to the new framework model for longitudinal data with the typically sparse and irregular designs. The non-smooth quantile loss and functional linear operator pose new challenges to functional data analysis for longitudinal data in both computation and theoretical development. To address the challenge, we propose the iterative surrogate least squares estimation approach for the FLOQR model, which transforms the response trajectories and establishes a new connection between FLOQR and functional linear operator model. In addition, we use Karhunen–Loève expansion to alleviate the problem of the nonexistence of the inverse of the covariance in the infinite-dimensional Hilbert space. Then, the approach is used to classic functional varying coefficient QR, functional linear QR, and functional varying coefficient QR with history index function for sparse longitudinal data by using functional principal components analysis through conditional expectation. The resulting technique is flexible and allows the prediction of an unobserved quantile response trajectory from sparse measurements of a predictor trajectory. Theoretically, we show that, after a constant number of iterations, the proposed estimator is asymptotic consistent for sparse designs. Moreover, asymptotic pointwise confidence bands are obtained for predicted quantile individual trajectories based on their asymptotic distributions. The proposed algorithms perform well in simulations, and are illustrated with longitudinal primary biliary liver cirrhosis data.

Key words and phrases: Confidence band, functional data analysis, linear operator, prediction, quantile regression, repeated measurements, sparse design.

1. Introduction

Functional data analysis has become increasingly useful in various fields. One well-known application is growth curve analysis in biology, medicine, and chemistry; see, for instance, Müller (2009), Ramsay and Silverman (2005), Ferraty and Vieu (2006), and references therein. In these applications, especially in longitudinal studies, often a few repeated measurements can be obtained for each subject or item, owing to cost or logistical constraints that limit the number

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