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ESTIMATION AND GOODNESS-OF-FIT TESTING FOR NON-NEGATIVE RANDOM VARIABLES WITH EXPLICIT LAPLACE TRANSFORM

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Abstract: Many flexible families of positive random variables exhibit non-closed forms of the density and distribution functions and this feature is considered unappealing for modelling purposes. However, such families are often characterized by a simple expression of the corresponding Laplace transform. Relying on the Laplace transform, we propose to carry out parameter estimation and goodness-of-fit testing for a general class of non-standard laws. We suggest a novel data-driven inferential technique, providing parameter estimators and goodness-of-fit tests, whose large-sample properties are derived. The implementation of the method is specifically considered for the positive stable and Tweedie distributions. A Monte Carlo study shows good finite-sample performance of the proposed technique for such laws.

Key words and phrases: Central limit theorem, consistent estimation, goodness-of-fit testing, Laplace transform, stable distribution, Tweedie distribution.

1. Introduction

Large classes of positive random variables display a simple closed form of the Laplace transform, even if their density functions can be solely given by means of special functions or series, which eventually require rather complex algorithms for their computation. Owing to this shortcoming, field scientists often discard the use of such random variables, although they are appropriate for modelling real data.

An archetype of such a class of laws is the positive stable distribution, which may be very suitable to model data with Paretian tails (Nolan, 2020). The density function of a positive stable random variable can be expressed by means of the Wright function (Barabesi, 2020), which is unfortunately awkward to compute; see the algorithms proposed by Luchko (2008). To this aim, Barabesi (2020), Dunn and Smyth (2005), and Dunn and Smyth (2008) suggest some approximation methods based on *ad-hoc* Fourier or Laplace inversion techniques. However, these algorithms can be time-consuming and possibly inadequate for

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