

FUNCTIONAL VARYING-COEFFICIENT MODEL UNDER HETEROSKEDASTICITY WITH APPLICATION TO DTI DATA

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Abstract: In this paper, we develop a multi-step estimation procedure to simultaneously estimate the varying-coefficient functions using a local-linear generalized method of moments (GMM) based on continuous moment conditions. To incorporate spatial dependence, the continuous moment conditions are first projected onto eigen-functions and then combined by weighted eigen-values, thereby, solving the challenges of using an inverse covariance operator directly. We propose an optimal instrument variable that minimizes the asymptotic variance function among the class of all local-linear GMM estimators, and it outperforms the initial estimates which do not incorporate the spatial dependence. Our proposed method significantly improves the accuracy of the estimation under heteroskedasticity and its asymptotic properties have been investigated. Extensive simulation studies illustrate the finite sample performance, and the efficacy of the proposed method is confirmed by real data analysis.

Key words and phrases: Diffusion tensor imaging, heteroskedasticity, local-linear GMM, moment conditions, multi-step estimation procedure, varying-coefficient model.

1. Introduction

Due to modern advancements in technology, varying-coefficient models in functional data have become popular to analyze data coming from several imaging technologies such as magnetic resonance imaging (MRI), diffusion tensor imaging (DTI), etc. We consider the problem of estimating non-parametric coefficient function $\beta(s)$ which is defined on the functional domain (for example, space) $\mathcal{S}$ to understand the relationship between functional response $Y(s)$ and real-valued covariates denoted by $\mathbf{X} = (X_1, \dots, X_p)^T$, which takes the following form,

$$Y(s) = \mathbf{X}^T \beta(s) + U(s), \quad (1.1)$$

where $\beta(s) = \{\beta_1(s), \dots, \beta_p(s)\}^T$ is a p -dimensional vector of unknown smooth functions, and it is assumed that $\beta(\cdot)$ is twice-differentiable with continuous second-order derivatives. The random error $\{U_i(s) : s \in \mathcal{S}\}$ is assumed to be a stochastic process indexed by $s \in \mathcal{S}$ and it characterizes the within-curve

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