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# CONCENTRATION INEQUALITIES FOR HIGH-DIMENSIONAL LINEAR PROCESSES WITH DEPENDENT INNOVATIONS

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*Abstract:* We develop concentration inequalities for the $l_\infty$ norm of vector linear processes with sub-Weibull, mixingale innovations. This inequality is used to obtain a concentration bound for the maximum entrywise norm of the lag-$h$ autocovariance matrix of linear processes. We apply these inequalities to sparse estimation of large-dimensional VAR(p) systems and heterocedasticity and autocorrelation consistent (HAC) high-dimensional covariance estimation.

*Key words and phrases:* Autocovariance, HAC, high-dimensional time series, linear process, mixingale, sub-Weibull.

## 1. Introduction

In this paper we develop general concentration inequalities that are used in high-dimensional statistical literature. We study a class of high-dimensional time series that can be represented by linear processes with dependent innovations. Specifically, we assume that innovation process is mixingale with sub-Weibull tails. This specification covers a wide range of data-generating processes, such as factor and conditionally heteroskedastic models, as discussed in Wong, Li and Tewari (2020), Bours and Steland (2021), Wilms et al. (2021), Masini, Medeiros and Mendes (2022), and Adamek, Smeekes and Wilms (2023a), among many others.

Linear processes are widely used in time series analysis for their ability to represent a wide range of dependent processes. For example, the Wold decomposition theorem represents stationary nonlinear processes as a linear process with uncorrelated innovations. The Vector Autoregressive Moving Average (VARMA) model, in turn, approximates the time series by a linear process indexed by a finite number of parameters. Typically, innovations are assumed to be independent, allowing for well-understood asymptotic properties (Hall and Heyde, 1980; Phillips and Solo, 1992; Lütkepohl, 2006). As an alternative to independence, Wang, Lin and Gulati (2001) shows the weak convergence of partial sums for martingale difference innovations, while Wu and Min (2005) establish a central limit theorem and the invariance principle for

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