

ASYMPTOTIC RESULTS FOR PENALIZED QUASI-LIKELIHOOD ESTIMATION IN GENERALIZED LINEAR MIXED MODELS

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Abstract: Generalized Linear Mixed Models (GLMMs) are widely used for analysing clustered data. One well-established method of overcoming the integral in the marginal likelihood function for GLMMs is penalized quasi-likelihood (PQL) estimation, although to date there are few asymptotic distribution results relating to PQL estimation for GLMMs in the literature. In this paper, we establish large-sample results for PQL estimators of the parameters and random effects in independent-cluster GLMMs, when both the number of clusters and the cluster sizes go to infinity. This is done under two distinct regimes: conditional on the random effects (essentially treating them as fixed effects) and unconditionally (treating the random effects as random). Under the conditional regime, we show the PQL estimators are asymptotically normal around the true fixed and random effects. Unconditionally, we prove that while the estimator of the fixed effects is asymptotically normally distributed, the correct asymptotic distribution of the so-called prediction gap of the random effects may in fact be a normal scale-mixture distribution under certain relative rates of growth. A simulation study is used to verify the finite-sample performance of our theoretical results.

Key words and phrases: Asymptotic independence, clustered data, large-sample distribution, longitudinal data, prediction.

1. Introduction

Generalized linear mixed models (GLMMs) are widely used in statistics to model relationships in clustered and correlated data (McCulloch and Searle, 2004). As the marginal likelihood function of GLMMs, except for normally distributed responses with the identity link, contains an intractable integral, many methods have been developed to estimate and perform inference for the parameters in a computationally efficient manner. These include the Laplace approximation, Gauss–Hermite quadrature, and variational approximations, among others (McCulloch and Searle, 2004; Ormerod and Wand, 2012; Brooks et al., 2017). A connected and well-established approach is penalized Quasi-Likelihood (PQL) estimation (Breslow and Clayton, 1993). As one of the first methods to circumvent the intractable integral, PQL estimation has seen a resurgence in

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