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# STATISTICAL INFERENCE FOR LOCAL GRANGER CAUSALITY

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*Abstract:* Granger causality has been employed to investigate causality relations between components of stationary multiple time series. We generalize this concept by developing statistical inference for local Granger causality for multivariate locally stationary processes. Our proposed local Granger causality approach captures time-evolving causality relationships in nonstationary processes. The proposed local Granger causality is well represented in the frequency domain and estimated based on the parametric time-varying spectral density matrix using the local Whittle likelihood. Under regularity conditions, we demonstrate that the estimators converge to multivariate normal in distribution. Additionally, the test statistic for the local Granger causality is shown to be asymptotically distributed as a quadratic form of a multivariate normal distribution. For practical demonstration, the proposed local Granger causality method uncovered new functional connectivity relationships between channels in brain signals. Moreover, the method was applicable to topological data analysis to identify structural changes in financial data.

*Key words and phrases:* Brain signals, local Granger causality, local Whittle likelihood, multivariate locally stationary processes, time-varying spectral density matrix, topological data analysis.

## 1. Introduction

Statistical inference for cause and effect remains at the forefront of many studies including biology, medicine, physical systems, environmental science, public health, policy and finance. However, there remain challenges on inference because causality is notoriously difficult to establish. Granger causality, proposed in Granger (1963, 1969), is a milestone of causal inference in dynamic models. In broad terms, Granger causality from a time series $\{Y_t\}$ to another series $\{X_t\}$ measures the predictive ability from the series $\{Y_t\}$ to $\{X_t\}$. If the predictive ability of $(X_s, Y_s)_{s<t}$ on $X_t$ is not different from the predictive ability of $(X_s)_{s<t}$ on $X_t$, then there is "no Granger causal relationship" from the series $\{Y_t\}$ to $\{X_t\}$. Thus, Granger causality analysis is important for determining whether or not a set of variables contains useful information for improving the prediction of another set of variables. Measures of linear dependence and feedback between components of a multivariate time series in both time and frequency domains

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