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IDENTIFIABILITY AND ESTIMATION OF CAUSAL EFFECTS WITH NON-GAUSSIANITY AND AUXILIARY COVARIATES

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Abstract: Assessing causal effects in the presence of unmeasured confounding is challenging. Although auxiliary variables, such as instrumental variables, are commonly used to identify causal effects, they are often unavailable in practice due to stringent and untestable conditions. To address this issue, previous researches have utilized linear structural equation models to show that the causal effect is identifiable when noise variables of the treatment and outcome are both non-Gaussian. In this paper, we investigate the problem of identifying the causal effect using the auxiliary covariate and non-Gaussianity from the treatment. Our key idea is to characterize the impact of unmeasured confounders using an observed covariate, assuming they are all Gaussian. We demonstrate that the causal effect can be identified using a measured covariate, and then extend the identification results to the multi-treatment setting. We further develop a simple estimation procedure for estimating causal effects and derive a $\sqrt{n}$-consistent estimator. Finally, we evaluate the performance of our estimator through simulation studies and apply our method to investigate the effect of the trade on income.

Key words and phrases: Auxiliary variable, causal effects, identification, multiple treatments, non-Gaussianity.

1. Introduction

Identifying and estimating the causal effect of a treatment on an outcome is crucial in practice, as it provides insight into the effectiveness of a given intervention. However, the existence of unmeasured confounding may be a core issue in observational studies. A common assumption of causal inference using observational data is exchangeability, which requires that one has measured a sufficiently rich set of covariates (Rosenbaum and Rubin, 1983). This is often challenged because investigators usually cannot accurately learn the confounding mechanism from the measured covariates in real scenarios.

Numerous strategies have been proposed for addressing identification issue of causal effects under unmeasured confounding. With a valid instrumental variable (IV) that satisfies relevance, independence, and exclusion restriction assumption, the causal effect can be identified for a binary treatment

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