

# LOCALLY OPTIMAL DESIGNS FOR ESTIMATING ONE OR MORE FUNCTIONS OF SHARED PARAMETERS BETWEEN TWO GROUPS IN BIOMEDICAL STUDIES

Xin Liu<sup>1</sup>, Rong-Xian Yue<sup>2,3</sup> and Weng Kee Wong<sup>\*4</sup>

<sup>1</sup>*Donghua University*, <sup>2</sup>*Fuyao University of Science and Technology*,

<sup>3</sup>*Shanghai Normal University and*

<sup>4</sup>*University of California, Los Angeles*

*Abstract:* Models with shared parameters arise quite naturally in the biological sciences and we use optimal design theory to construct  $c$ -optimal approximate designs for estimating one or more functions of the model parameters in two regression models with shared parameters. We assume sample sizes for the two groups are fixed and establish equivalence theorems to confirm the optimality of the design. As applications, we consider the parallel dose response model, the EMAX model and the Exponential model, each with shared parameters. The methodology is general and can be applied to other models or design problems. For example, we show the theoretical framework can be directly extended to the case when we are interested to find a  $c$ -optimal design to estimate the mean difference between the expected responses at an extrapolated dose for a nonlinear model, or when the total sample size for the whole study is fixed, and we wish to determine the optimal proportions of observations to allocate to the two groups, or we have multivariate responses.

*Key words and phrases:* Approximate design, equivalence theorem, group comparison,  $L$ -optimal design.

## 1. Introduction

Group comparison is a basic and important problem across disciplines. A statistical model is usually used to model the outcome from the two or more groups and assess group differences. Statistical models with shared parameters are increasingly common and they arise quite naturally in various research areas. For instance, Chakraborty et al. (2016) employed dynamic regimens trials for precision medicine in a depression clinical trial using models with shared parameters. Models with shared parameters are also increasingly used in pharmaceutical studies, where experts believe that some parameters in the drug profiles have the same values when the drugs are in the same class.

There is some research work on constructing  $D$ -optimal designs analytically for a two-group experiments with shared parameters; see for example, Feller et al. (2017).  $D$ -optimal designs minimize the generalized variance of all the

---

\*Corresponding author. E-mail: [wkwong@ucla.edu](mailto:wkwong@ucla.edu)