Statistica Sinica **36** (2026), 1-21
doi:https://doi.org/10.5705/ss.202023.0268

INTRINSIC MINIMUM AVERAGE VARIANCE ESTIMATION FOR DIMENSION REDUCTION WITH SYMMETRIC POSITIVE-DEFINITE MATRICES AND BEYOND

Baiyu Chen, Shuang Dai and Zhou Yu*

East China Normal University

Abstract: In this paper, we estimate the central mean subspace in a dimension reduction problem where the response is a symmetric positive-definite matrix. We propose the intrinsic minimum average variance estimation and the intrinsic outer product of gradient method which fully exploit the geometric structure of the Riemannian manifold where the response resides. We present algorithms for our newly developed methods under the log-Euclidean metric and the log-Cholesky metric. The two metrics endow the manifold with a commutative Lie group structure that transforms our manifold model into a Euclidean one and helps us derive the consistency and asymptotic normality of estimators. Our methods are then naturally extended to the case allowing $p = p_n$ to diverge and the case of general Riemannian manifolds. Several simulation studies and an application to the New York taxi network data showcase the superiority of our proposals.

Key words and phrases: Central mean subspace, log-Cholesky metric, log-Euclidean metric, minimum average variance estimation, outer product of gradient, symmetric positive-definite matrix.

1. Introduction

For $Y \in R$ and $X \in R^p$, sufficient dimension reduction (SDR) seeks a $p \times d$ matrix B with $d << p$ such that $Y \perp\!\!\!\perp X \mid B^\mathsf{T} X$. The space spanned by the columns of B, denoted by $\mathcal{S}(B)$, is called an SDR subspace. If $\mathcal{S}(B)$ is a subspace of all other SDR subspaces, it is called the central subspace (CS). Popular methods estimating CS include sliced inverse regression (Li, 1991), sliced average variance estimation (Cook and Weisberg, 1991), directional regression (Li and Wang, 2007), semiparametric approaches (Ma and Zhu, 2012, 2013, 2019), among others. Although CS provides a complete picture of the dependency of Y on X, one might be only interested in the conditional mean function for which the dimension reduction assumes

$$Y \perp\!\!\!\perp E(Y \mid X) \mid B^\mathsf{T} X. \tag{1.1}$$

*Corresponding author. E-mail: zyu@stat.ecnu.edu.cn