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SIMULTANEOUS INFERENCE FOR THE DISTRIBUTION OF FUNCTIONAL PRINCIPAL COMPONENT SCORES

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Abstract: This paper introduces a novel methodology for simultaneous inference of the cumulative distribution function (CDF) of functional principal component (FPC) scores. We establish a general framework for estimating the CDF, including both nonsmooth and smooth estimators, and demonstrate their asymptotic equivalence. For dense functional data, we employ nonparametric pre-smoothing, ensuring oracle properties that make our estimators equivalent to those from fully observed trajectories. We recommend B-spline smoothing for its computational efficiency. Additionally, we derive theoretical properties to construct simultaneous confidence bands (SCBs) and develop new testing procedures for the distribution of FPC scores. These procedures, including Kolmogorov-Smirnov and Cramér-von Mises tests, can handle a diverging number of components and are particularly effective for testing the normality of functional data, a common assumption in literature and practice. Our methodology is supported by extensive numerical simulations and applied to well-known functional datasets and Electroencephalogram (EEG) data.

Key words and phrases: Cumulative distribution function, functional principal component scores, goodness of fit tests, nonparametric smoothing, Simultaneous inference.

1. Introduction

Functional data analysis, a key area in statistics applied in finance, environment, and neuroscience (Bosq, 2000; Hsing and Eubank, 2015), deals with variables as random functions over a domain. Despite the infinite dimensionality of these functions offering rich information, it poses challenges in pattern identification and signal extraction. Functional principal component analysis (FPCA), as detailed in Yao, Müller and Wang (2005b), Berkes et al. (2009), and Zhang et al. (2011), is crucial in dimension reduction and is widely used in functional data analysis, including functional linear models and change point analysis.

While there is extensive research on the eigenvalues and eigenfunctions of functional data (Dauxois, Pousse and Romain, 1982; Hall and Hosseini-Nasab, 2009; Kokoszka and Reimherr, 2013; Cai and Hu, 2024b, 2025b), statistical inference of functional principal component (FPC) scores remains underexplored.

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