Statistica Sinica **36** (2026), 1-22
doi:https://doi.org/10.5705/ss.202023.0238

TESTING FOR VARIANCE CHANGES UNDER VARYING MEAN AND SERIAL CORRELATION

Cheuk Wai Dominic Leung and Kin Wai Chan*

The Chinese University of Hong Kong

Abstract: Detection of variance change points is statistically difficult when the data exhibit a varying mean structure and autocorrelation. Existing variance change point tests either require the assumption of mean constancy or sacrifice testing power due to serial dependence. This article addresses these problems by proposing a trend-robust and autocorrelation-efficient variance change point test via a differencing approach. This approach removes the mean effect without fitting the mean function. It also allows the test to retrieve the reduced power due to serial dependence. We prove that the optimal difference-based test should minimize the long-run coefficient of variation of the sample second moment of the noise instead of the long-run variance in the presence of serial dependence. The optimal solution can be efficiently computed by fractional quadratic programming. The asymptotic relative efficiency under a local alternative hypothesis is derived. A rate-optimal long-run variance estimator is also proposed. It is proven to be doubly robust against varying mean and variance change points.

Key words and phrases: Change point, cumulative sum, difference sequence, long-run variance, non-linear time series.

1. Introduction

Abrupt variance changes provide insights that cannot be explained by the variability of the means. Applications cover various fields, e.g., finance (Inclan and Tiao, 1994), environmental science (Gerstenberger, Vogel and Wendler, 2020), medical science (Gao et al., 2019); see also Hsu, Miller and Wichern (1974), Lee and Park (2001), Lee, Na and Na (2003), and Aue et al. (2009). In this article, we assume that the observed time series $X_1, \ldots, X_N$ are generated as follows:

$$X_i = \mu_i + \sigma_i Z_i \qquad (i = 1, \ldots, N), \tag{1.1}$$

where $\mu_1, \ldots, \mu_N$ are possibly non-constant deterministic signals, $\sigma_1, \ldots, \sigma_N$ are deterministic marginal standard deviations, and $(Z_i)_{i\in\mathbb{Z}}$ is a zero-mean unit-variance strictly stationary noise time series. Our goal is to test $H_0 : \sigma_1 = \cdots = \sigma_N$ against

$$H_1 : \sigma_0 \equiv \sigma_1 = \cdots = \sigma_{k^\star} \neq \sigma_{k^\star+1} = \cdots = \sigma_N \equiv \sigma_0 e^{\Delta} \tag{1.2}$$

*Corresponding author. E-mail: kinwaichan@cuhk.edu.hk