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OPTIMAL PRIORS FOR THE DISCOUNTING PARAMETER OF THE NORMALIZED POWER PRIOR

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Abstract: The power prior is a popular class of informative priors for incorporating information from historical data. It involves raising the likelihood for the historical data to a power, which acts as discounting parameter. When the discounting parameter is modelled as random, the normalized power prior is recommended. In this work, we prove that the marginal posterior for the discounting parameter for generalized linear models converges to a point mass at zero if there is any discrepancy between the historical and current data, and that it does not converge to a point mass at one when they are fully compatible. In addition, we explore the construction of optimal priors for the discounting parameter in a normalized power prior. In particular, we are interested in achieving the dual objectives of encouraging borrowing when the historical and current data are compatible and limiting borrowing when they are in conflict. We propose intuitive procedures for eliciting the shape parameters of a beta prior for the discounting parameter based on two minimization criteria, the Kullback–Leibler divergence and the mean squared error. Based on the proposed criteria, the optimal priors derived are often quite different from commonly used priors such as the uniform prior.

Key words and phrases: Bayesian analysis, clinical trial, normalized power prior, power prior.

1. Introduction

The power prior (Ibrahim and Chen, 2000) is a popular class of informative priors that allow the incorporation of historical data through a tempering of the likelihood. It is constructed by raising the historical data likelihood to a power a_0, where $0 \leq a_0 \leq 1$. The discounting parameter a_0 can be fixed or modelled as random. When it is modelled as random and estimated jointly with other parameters of interest, the normalized power prior (NPP) (Duan, Ye and Smith, 2006) is recommended as it appropriately accounts for the normalizing function necessary for forming the correct joint prior distribution (Neuenschwander, Branson and Spiegelhalter, 2009). Many extensions of the power prior and the normalized power prior have been developed. Banbeta et al.

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