

VERSATILE PARAMETRIC CLASSES OF COVARIANCE FUNCTIONS THAT INTERLACE ANISOTROPIES AND HOLE EFFECTS

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Abstract: Covariance functions are a fundamental tool for modeling the dependence structure of spatial random fields. This work investigates novel constructions for covariance functions that enable the integration of anisotropies and hole effects in complex and versatile ways, having the potential to provide more accurate representations of dependence structures arising with real-world data. We show that these constructions extend widely used covariance models, including the Matérn, Cauchy, compactly-supported hypergeometric and cardinal sine models. We apply our results to a geophysical data set from a rock-carbonate aquifer and demonstrate that the proposed models yield more accurate predictions at unsampled locations compared to basic covariance models.

Key words and phrases: Anisotropic random fields, Cauchy covariance, cardinal sine covariance, Gauss hypergeometric covariance, Matérn covariance, Nonmonotonic covariance models.

1. Introduction

Data indexed by spatial (hereafter, Euclidean) coordinates arise in many disciplines of the natural sciences, including climatology (Sang, Jun and Huang, 2011), oceanography (Wikle et al., 2013), environment science (Rodrigues et al., 2015), ecology (Finley, Banerjee and MacFarlane, 2011), and geosciences (Davis, 2002). Statistical and geostatistical models often assume the observed data to be a realization of a Gaussian random field, with the covariance function being the fundamental ingredient to capture the spatial dependence (Chilès and Delfiner, 2012), to understand the underlying spatial patterns and make reliable predictions.

Currently, there is a fairly extensive catalog of parametric families of stationary covariance functions that allow modeling a large number of patterns appearing in real situations, such as long-memory, hole effects, periodicities, degree of mean square differentiability, anisotropies, among others. Classical textbooks, such as Gaetan and Guyon (2010) and Chilès and Delfiner (2012), provide extensive insights into the wide range of available models. While existing models can handle many common patterns found in real data sets, some data

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