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ESTIMATION OF A DISTRIBUTION WITH A BIAS AND ITS APPLICATIONS TO COMPETING RISKS

Hammou El Barmi* and Zhiliang Ying

The City University of New York and Columbia University

Abstract: A random variable X is said to have a symmetric distribution function (DF) about zero if X and $-X$ have the same distribution. The estimation of such a distribution and tests for symmetry are widely studied in the literature. Some of the alternatives to symmetry describe some notion of skewness or one-sided bias in terms of an ordering of the distributions of X and $-X$. One such ordering is characterized by $r_{-X}(x) \leq r_X(x)$ for all $x > 0$ where $r_{-X}(x)$ and $r_X(x)$ are the hazard rates of $-X$ and X, respectively. This is equivalent to the ratio $P(X > x)/P(X < -x)$ being nondecreasing in $x > 0$. In this paper we derive the nonparametric maximum likelihood estimator (NPMLE) of F under this constraint and show that it is inconsistent. We then construct a new estimator and establish its consistency and weak convergence. We also develop a test for symmetry against this one-sided alternative and study the finite sample performance of this new estimator. We show through simulations that it outperforms the NPMLE in terms of mean squared error for all the distributions under consideration. We also show how to apply this approach to compare the conditional distributions (conditional on the risks) of two competing risks in a competing risks model.

Key words and phrases: Competing risks, consistency, nonparametric likelihood estimator, restricted estimation, weak convergence.

1. Introduction

A random variable X with DF F is said to have a symmetric distribution about zero if X and $-X$ have the same distribution. Symmetry of the underlying distribution is a commonly occurring assumption in many statistical analyses and the validity of some of frequently used procedures depends heavily on this assumption. This is particularly the case for several nonparametric procedures such as the Wilcoxon signed rank test. It is also the case that many statistical procedures that are based on normality are generally robust to this assumption when the underlying distribution is symmetric (Chaffin and Rhiel, 1993). For these reasons, a large number of nonparametric tests for symmetry have been developed. Many of these tests are variations of the sign test, the Wilcoxon test, the Kolmogorov–Smirnov test or the Cramér–von Mises test (Shorack and Wellner, 1986). The simplest, and the most common alternatives are one-sided or two-sided shifts.

*Corresponding author. E-mail: hammou.elbarmi@baruch.cuny.edu