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BLIND SOURCE SEPARATION OVER SPACE: AN EIGENANALYSIS APPROACH

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Abstract: We propose a new estimation method for the blind source separation model of Bachoc et al. (2020). The new estimation is based on an eigenanalysis of a positive definite matrix defined in terms of multiple normalized spatial local covariance matrices, and therefore can handle moderately high-dimensional random fields. The consistency of the estimated mixing matrix is established with explicit error rates even when the eigen-gap decays to zero slowly. The proposed method is illustrated via both simulation and a real data example.

Key words and phrases: Eigen-analysis, eigen-gap, high-dimensional random field, mixing matrix, normalized spatial local covariance matrix.

1. Introduction

Blind source separation is an effective way to reduce the complexity in modelling p-variant spatial data (Nordhausen et al., 2015; Bachoc et al., 2020). The approach can be viewed as a version of independent component analysis (Hyvärinen, Karhunen and Oja, 2001) for multivariate spatial random fields. Though only the second moment properties are concerned, the challenge is to decorrelate p spatial random fields at the same location as well as across different locations. Note that the standard principal component analysis does not capture spatial correlations, as it only diagonalizes the covariance matrix (at the same location). Nordhausen et al. (2015) introduced a so-called local covariance matrix (see (2.4) in Sec. 2.2) to represent the dependence across different locations. Furthermore, it proposed to estimate the mixing matrix, defined in (2.1) in Section 2.1 below, in the blind source separation decomposition based on a generalized eigenanalysis, which can be viewed as an extension of the principal component analysis as it diagonalizes a local covariance matrix in addition to the standard covariance matrix. To overcome the drawback of using the information from only one local covariance matrix, Bachoc et al. (2020) proposed to use multiple local covariance matrices in the estimation (see (2.5) in Sec. 2.2). The method of Bachoc et al. (2020) has a clear advantage in incorporating the spatial dependence information over different ranges. It is in the spirit of JADE

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