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CONFIDENCE SURFACES FOR THE MEAN OF LOCALLY STATIONARY FUNCTIONAL TIME SERIES

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Abstract: The problem of constructing a simultaneous confidence surface for the two-dimensional mean function of a non-stationary functional time series is challenging as these bands cannot be built on classical limit theory for the maximum absolute deviation between an estimate and the time-dependent regression function. In this paper, we propose a new bootstrap methodology to construct such a region. Our approach is based on a Gaussian approximation for the maximum norm of sparse high-dimensional vectors approximating the maximum absolute deviation which is suitable for nonparametric inference of high-dimensional time series. The elimination of the zero entries produces (besides the time dependence) additional dependencies such that the "classical" multiplier bootstrap is not applicable. To solve this issue we develop a novel multiplier bootstrap, where blocks of the coordinates of the vectors are multiplied with random variables, which mimic the specific structure between the vectors appearing in the Gaussian approximation. We prove the validity of our approach by asymptotic theory, demonstrate good finite-sample properties by means of a simulation study and illustrate its applicability by analyzing a data example.

Key words and phrases: Confidence surface, functional data, Gaussian approximation, locally stationary time series, multiplier bootstrap.

1. Introduction

In the big data era data gathering technologies provide enormous amounts of data with complex structure. In many applications the observed data exhibits certain degrees of dependence and smoothness and thus may naturally be regarded as discretized functions. A major tool for the statistical analysis of such data is functional data analysis (FDA) which has found considerable attention in the statistical literature (see, for example, the monographs of Bosq, 2000; Ramsay and Silverman, 2005; Ferraty and Vieu, 2010; Horváth and Kokoszka, 2012; Hsing and Eubank, 2015, among others). In FDA the considered parameters, such as the mean or the (auto-)covariance (operator) are functions themselves, which makes the development of statistical methodology challenging. Most of the literature considers Hilbert space-based methodology for which there exists by now a well-developed theory. In particular, this approach allows the application of dimension reduction techniques such as (functional) principal components (see, for example,

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