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STATISTICAL INFERENCE FOR MEAN FUNCTIONS OF COMPLEX 3D OBJECTS

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Abstract: The use of complex three-dimensional (3D) objects is growing in various applications as data collection techniques continue to evolve. Identifying and locating significant effects within these objects is essential for making informed decisions based on the data. This article presents an advanced nonparametric method for learning and inferring complex 3D objects, enabling accurate estimation of the underlying signals and efficient detection and localization of significant effects. The proposed method addresses the problem of analyzing irregular-shaped 3D objects by modeling them as functional data and utilizing trivariate spline smoothing based on triangulations to estimate the underlying signals. We develop a highly efficient procedure that accurately estimates the mean and covariance functions, as well as the eigenvalues and eigenfunctions. Furthermore, we rigorously establish the asymptotic properties of these estimators. Additionally, a novel approach for constructing simultaneous confidence corridors to quantify estimation uncertainty is presented, and the procedure is extended to accommodate comparisons between two independent samples. The finite-sample performance of the proposed methods is illustrated through numerical experiments and a real-data application using the Alzheimer's Disease Neuroimaging Initiative database.

Key words and phrases: Complex object analysis, functional principal component analysis, localization, simultaneous confidence corridors, triangulation, trivariate splines.

1. Introduction

As data collection techniques continue to advance, complex three-dimensional (3D) objects are becoming more prevalent in new statistical applications. To effectively extract information from these objects, it is often necessary to utilize multiple images captured from different perspectives. These 3D images play a critical role in areas such as biomedical research, robotics, and engineering. For example, in medical imaging, 3D images are used to create detailed human body scans, such as magnetic resonance imaging (MRI), functional MRI, and positron emission tomography (PET) imaging. All these images can be used to diagnose and treat a variety of medical conditions. When working with complex objects,

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