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OPTIMAL DESIGNS FOR FUNCTIONAL PRINCIPAL AND EMPIRICAL COMPONENT SCORES

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Abstract: Sparse functional data analysis (FDA) is powerful for making inference on the underlying random function when noisy observations are collected at sparse time points. To have a precise inference, knowledge on optimal designs that allow the experimenters to collect informative functional data is crucial. Here, we propose a framework for selecting optimal designs to precisely predict functional principal and empirical component scores. Our work gives a relevant generalization of previous results on the design for predicting individual response curves. We obtain optimal designs, and evaluate the performance of commonly used designs. We demonstrate that without a judiciously selected design, there can be a great loss in statistical efficiency.

Key words and phrases: Design efficiency, exact designs, mixed model equations, random function, sparse orthonormal approximation.

1. Introduction

We are concerned with optimal experimental designs for sparse functional data analysis (FDA). With limited resources and practical constraints, it is common to have only a small number of observations from the underlying random function. Many sparse FDA methods are developed and shown powerful in analyzing such sparse functional data (Shi, Weiss and Taylor, 1996; Yao, Müller and Wang, 2005a,b; Hall, Müller and Wang, 2006; Hall, Müller and Yao, 2008; Müller, 2008; Nie et al., 2022; Zhong et al., 2022). However, knowledge on optimal designs for collecting informative sparse functional data remains scarce. For many sparse FDA methods, there is no guidance on the evaluation and selection of designs, and one may (randomly) select a design without having a good knowledge of its performance. As demonstrated in Ji and Müller (2017), and in this work, imprudently selected designs can result in a great loss in statistical efficiency, making the experiment inefficient, or even impossible, to achieve the study objectives of interest. The development of knowledge on optimal designs is crucial.

Due to its importance, the research on the design for sparse FDA has recently drawn some attention. As a pioneering work, Ji and Müller (2017) proposed optimal designs for two study objectives, including (i) trajectory recovery of the

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