

NONPARAMETRIC COMPARISONS OF MULTIPLE DISTRIBUTIONS UNDER UNIFORM STOCHASTIC ORDERING

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Abstract: Uniform stochastic ordering (USO), also known as hazard rate or failure rate ordering, has garnered significant interest across various applications. In this study, we present nonparametric approaches for comparing distributions within the framework of USO using the ordinal dominance curve. Our study consists of three main components. The first one offers new tests for equality among multiple distributions under USO assumptions. Secondly, we provide goodness-of-fit tests to investigate whether multiple distributions adhere to USO. Lastly, we identify distributions that exhibit significant statistical differences within the context of USO. We provide asymptotic properties and supporting numerical evidence for our proposed methods. To exemplify the application of our inferential techniques, we focus on a biomarker, microfibrillar-associated protein 4, and assess its potential for diagnosing fibrosis stages in hepatitis C patients.

Key words and phrases: Bonferroni correction, Brownian bridge, hazard rate ordering, order-restricted inference, ordinal dominance curve.

1. Introduction

Uniform Stochastic Ordering (USO) has held significant importance in various applications since it was introduced by Lehmann (1955). Define two independent random variables X_1 and X_2 with distributions F_1 and F_2 , respectively. Denoted by $X_1 \preceq X_2$ or $F_1 \preceq F_2$, we say that X_1 is smaller than X_2 in the sense of USO when the ratio $\{1 - F_1(t)\} / \{1 - F_2(t)\}$ is nonincreasing in t whenever $F_2(t) < 1$. When both F_1 and F_2 are absolutely continuous, USO is also known as hazard rate ordering. Applications in actuarial science, biology, economics, reliability, and survival analysis further emphasize the versatility of USO, e.g., see Dykstra, Kocher and Robertson (1991), Navarro and Shaked (2006), Da and Ding (2016), and Balakrishnan, Zhang and Zhao (2018). USO is stronger than ordinary stochastic ordering but weaker than likelihood ratio ordering (Keilson and Sumita, 1982; Shaked and Shanthikumar, 2007).

When data are collected from $k > 2$ distributions, a well-ordered USO, say $F_1 \preceq F_2 \preceq \cdots \preceq F_k$, has garnered considerable attention in the literature. In this work, we also focus on $F_1 \preceq F_2 \preceq \cdots \preceq F_k$ and examine three fundamental

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