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HOW SENSITIVE ARE TAIL-RELATED RISK MEASURES IN A CONTAMINATION NEIGHBOURHOOD?

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Abstract: Estimation or mis-specification errors in the portfolio return distribution can have a considerable impact on risk measures. This paper investigates the sensitivity of tail-related risk measures including the Value-at-Risk, expected shortfall and the expectile-quantile transformation level in an epsilon-contamination neighbourhood. The findings give the different approximations according to the tail heaviness of the contamination models and its contamination levels. Illustrating examples and an empirical study on the Royalton CRIX capturing and displaying the market movements are given. The codes used to obtain the results in this paper are available via QuantLet/SRMC.

Key words and phrases: Expected shortfall, expectile, risk measures, Royalton CRIX, sensitivity.

1. Introduction

Risk measures are used for both financial institutions' internal risk management and external regulation—for example, in the Basel Accord for risk-based requirements for regulatory capital (Chernobai, Rachev and Fabozzi, 2008). Both academics and practitioners are devoted to developing appropriate risk measures with good properties, including robustness, elicitability, and backtesting (He, Kou and Peng, 2022; Gneiting, 2011). Note that risk measures are defined as functionals of the unknown portfolio loss distributions, and the particular difficulty in measuring risk is that the tail part of a loss distribution bears substantial model uncertainty. On the other hand, estimation or mis-specification errors in the portfolio loss distribution can have a considerable impact on risk measures, and it is important to examine the robustness of risk measures to these errors (Bernard, Pesenti and Vanduffel, 2024; Cont, Deguest and Scandolo, 2010). In this work, we focus on the three most common risk measures, Value-at-Risk, expectile and expected shortfall, and examine how the tail of these risk measures varies in the contamination field in terms of Huber (1964).

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