Statistica Sinica **35** (2025), 1-22
doi:https://doi.org/10.5705/ss.202022.0404

A NEW PARADIGM FOR GENERATIVE ADVERSARIAL NETWORKS BASED ON RANDOMIZED DECISION RULES

Sehwan Kim, Qifan Song and Faming Liang*

Purdue University

Abstract: The Generative Adversarial Network (GAN) was recently introduced in the literature as a novel machine learning method for training generative models. It has many applications in statistics such as nonparametric clustering and nonparametric conditional independence tests. However, training the GAN is notoriously difficult due to the issue of mode collapse, which refers to the lack of diversity among generated data. In this paper, we identify the reasons why the GAN suffers from this issue, and to address it, we propose a new formulation for the GAN based on randomized decision rules. In the new formulation, the discriminator converges to a fixed point while the generator converges to a distribution at the Nash equilibrium. We propose to train the GAN by an empirical Bayes-like method by treating the discriminator as a hyper-parameter of the posterior distribution of the generator. Specifically, we simulate generators from its posterior distribution conditioned on the discriminator using a stochastic gradient Markov chain Monte Carlo (MCMC) algorithm, and update the discriminator using stochastic gradient descent along with simulations of the generators. We establish convergence of the proposed method to the Nash equilibrium. Apart from image generation, we apply the proposed method to nonparametric clustering and nonparametric conditional independence tests. A portion of the numerical results is presented in the Supplementary Material.

Key words and phrases: Generative model, minimax game, stochastic approximation, stochastic gradient Markov chain Monte Carlo.

1. Introduction

The Generative Adversarial Network (GAN) (Goodfellow et al., 2014) provides a novel way for training generative models which seek to generate new data with the same statistics as the training data. Other than image generation, the GAN has been used in many nonparametric statistical tasks, such as clustering (Mukherjee et al., 2019), conditional independent test (Bellot and van der Schaar, 2019), and density estimation (Singh et al., 2018; Liu et al., 2021). In this paper, we call the training data real samples, and those generated by the GAN fake samples.

In its original design, the GAN is trained by competing two neural networks, namely generator and discriminator, in a game. However, due to the instability

*Corresponding author. E-mail: fmliang@purdue.edu