Statistica Sinica **35** (2025), 1-25
doi:https://doi.org/10.5705/ss.202022.0382

RANK BASED TESTS FOR HIGH DIMENSIONAL WHITE NOISE

Dachuan Chen[#], Fengyi Song[#] and Long Feng[*]

Nankai University

Abstract: The development of high-dimensional white noise test is important in both statistical theories and applications, where the dimension of the time series can be comparable to or exceed the length of the time series. This paper proposes several distribution-free tests using the rank based statistics for testing the high-dimensional white noise, which are robust to the heavy tails and do not quire the finite-order moment assumptions for the sample distributions. Three families of rank based tests are analyzed in this paper, including the simple linear rank statistics, non-degenerate U-statistics and degenerate U-statistics. The asymptotic null distributions and rate optimality are established for each family of these tests. Among these tests, the test based on degenerate U-statistics can also detect the non-linear and non-monotone relationships in the autocorrelations. Moreover, this is the first result on the asymptotic distributions of rank correlation statistics which allowing for the cross-sectional dependence in high dimensional data.

Key words and phrases: Degenerate U-statistics, high dimensionality, non-degenerate U-statistics, serial correlation, simple linear rank statistics, white noise test.

1. Introduction

The hypothesis test for white noise is a critical methodology in statistical inference and modeling. It is necessary in diagnosis checking for the linear regression and time series modeling. There has been a vast increase in the amount of high-dimensional data available in recent years, which has received increasing attention from statisticians. The existence of such high-dimensional data is widespread, including the areas of genomics, neuroscience, finance, economics and so on. This brings additional difficulties for the problem of diagnosis checking, which means that in the theoretical development of the test for high-dimensional white noise, the dimension of the time series can be comparable to or exceed the length of the time series.

For the white noise tests designed for univariate time series, many commonly used methodologies are well documented in Li (2004). The alternative hypothesis of these tests can be grouped into two different classes: (i) specified alternative in form of some explicit parametric model; (ii) completely unspecified alternative,

[#]Co-first authors and contributed equally to this work.
[*]Corresponding author. E-mail: flnankai@nankai.edu.cn