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TEST OF PARTIAL SEPARABILITY FOR MULTIVARIATE FUNCTIONAL DATA

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Abstract: For multivariate functional data, it is quite challenging to model the cross-covariance structure which consists of dual aspects of multivariate and functional features. To simplify the cross-covariance analysis, the assumption of partial separability is widely used to decompose the data into an additive form of multivariate random variables and functional components. In this article, we propose hypothesis testing procedures to examine the validity of partial separability. We study the asymptotic properties of the l^2 and l^∞ norm of the test statistic, resulting in a chi-square type mixture test and a high-dimensional test that are suitable to finite- or high-dimensional multivariate functional data with diverse multivariate dependence. We assess the empirical performance of the proposed tests through two simulation studies for multivariate functional data and graphical functional data, followed by two corresponding real examples: multichannel tonnage data and electroencephalography data.

Key words and phrases: Functional graphical model, high-dimensional test, multivariate functional data, partial separability.

1. Introduction

The knowledge of the covariance structure plays an important role in functional data analysis (FDA). To characterize the variation of random curves, nonparametric methods are widely used, often coupled with dimension reduction tools such as functional principal component analysis (FPCA), which provides a flexible approach to depict the temporal dependence. Due to rapid developments in data collection techniques, multivariate functional data (MFD) that comprise simultaneous recordings across multiple processes are becoming increasingly available. Typical examples include daily traffic measurements (Chiou, Chen and Yang, 2014), temperature recordings (Berrendero, Justel and Svarc, 2011), multichannel profile data (Paynabar, Zou and Qiu, 2016) and neuroimaging data (Happ and Greven, 2018; Qiao, Guo and James, 2019). Advanced modeling techniques on the cross-covariance function/kernel/operator, which jointly depict the functional features across time and multivariate dependence across processes, are consequently attracting growing attention.

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