A COMMUNITY HAWKES MODEL FOR CONTINUOUS-TIME NETWORKS WITH INTERACTION HETEROGENEITY

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Abstract: Continuous-time networks have attracted significant attention due to their widespread applications in various disciplines. A rich literature considers the community structure of the nodes, while few have accounted for the node heterogeneity of interaction propensities. To simultaneously account for both the self-exciting feature and the node heterogeneity, we propose a model based on the Hawkes process, which allows the interaction intensity to vary flexibly with incurred nodes and their affiliated communities. We derive the likelihood function using the immigration-birth representation of the Hawkes process and develop an innovative expectation-maximization algorithm with membership refinement to tackle the computational challenge. Further, we establish the consistency of parameter estimation under mild assumptions. The effectiveness of our model is validated by extensive simulation studies on synthetic data as well as two real-world applications.

Key words and phrases: Community structure, dynamic network, EM algorithm, Hawkes process, node heterogeneity.

1. Introduction

Networks, especially dynamic networks, have attracted enormous research interest recently. In dynamic networks, nodal linkages are not fixed but may appear or disappear over time. Dynamic networks can be further divided into two types: discrete-time and continuous-time networks. The difference between these two types lies in the fact that the time points of interactions are either continuous-valued or discrete-valued. A crucial task of network analysis is understanding its generative mechanism. Specifically, we are interested in the linking probability of node pairs for static networks and the interaction frequency for dynamic networks. Statistical models inferred from these networks can then be used to discover existing generation patterns, to predict future links or to generate synthetic but realistic networks.

There has been significant research on static network generative models. The simplest model may be the Erdős–Rényi model in which all nodes are considered homogeneous (Erdős and Rényi, 1959, 1960). A natural extension accounting

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