Statistica Sinica **35** (2025), 1-24
doi:https://doi.org/10.5705/ss.202022.0312

ENHANCED STRUCTURAL BREAK DETECTION IN FUNCTIONAL MEANS

Shuhao Jiao*[1], Ngai Hang Chan[1,2] and Chun Yip Yau[2]

[1] *City University of Hong Kong and*
[2] *The Chinese University of Hong Kong*

Abstract: A new change-point detector for structure breaks in functional means is developed in this paper. The detector is based on a novel easy-to-implement approach of dimension reduction. One major advantage of the proposed method is its efficiency in selecting the basis functions that capture the change/jump of functional means, leading to a higher detection power. We thoroughly investigate the asymptotic properties of the proposed detector when both the sample size and the incorporated dimension increase. The numerical simulation studies justify the superiority of the proposed approach compared to the existing competitors and highlight the necessity of aligning the basis functions with the change to be detected. An application to annual humidity trajectories illustrates the practical superiority of the developed approach.

Key words and phrases: Change alignment, change point analysis, dimension reduction, functional mean, weakly dependent functional data.

1. Introduction

This paper provides a new method to tackle a popular problem in functional data analysis, detecting the change point in functional means of a sequence of functional time series. The general setting is that a single change point partitions the entire sequence into two local stationary blocks, where the functions in each block share the same mean function.

There have been a number of methods developed for functional structural breaks in mean function. Many of them are developed based on dimension reduction or projection. A typical step in projection-based approaches is to project the functions onto a finite number of basis functions, and the projection scores are employed to detect the change in mean of independent or dependent functional data. See, for example, Berkes et al. (2009), Aue et al. (2009), Zhang et al. (2011), Aston and Kirch (2012a), and the references therein. More recently, Fremdt et al. (2014) consider structural break detection by using functional principal component analysis (FPCA) with an increasing number of projections. Dimension reduction is also utilized to detect change points of multivariate functions under separability assumptions (e.g., spatial temporal data or brain

*Corresponding author. E-mail: shuhao.jiao@cityu.edu.hk