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ASYMMETRIC ESTIMATION FOR VARYING-COEFFICIENT ADDITIVE MODEL WITH FUNCTIONAL RESPONSE IN REPRODUCING KERNEL HILBERT SPACE

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Abstract: Function-on-scalar regression models are extensively utilized in applications involving longitudinal or functional responses. Prior literature has established the minimax optimal bounds for both mean and quantile regression. This paper explores expectile regression as a natural extension to mean regression, particularly for modeling potential heteroscedasticity in data. We propose an expectile function-on-scalar regression model that focuses on asymmetrical regression of functional responses based on scalar predictors. Employing the structure of Reproducing Kernel Hilbert Space (RKHS), we have developed a statistically efficient expectile estimator. This estimator comes with theoretical backing, derived from the minimax rates of convergence in both random and fixed design contexts. Our extensive simulations demonstrate the robust performance of the proposed methods across various settings. Additionally, we present an empirical analysis using quality of life data from a breast cancer clinical trial, showcasing the practical utility of our method.

Key words and phrases: Additive model, cancer clinical trials, functional regression, quality of life, reproducing kernel Hilbert space.

1. Introduction

Functional data analysis (FDA) has drawn significant attention from researchers from various fields due to its unique ability to deal with intrinsically infinite-dimensional data and processes defined on continuous domains. Data with such structure are widely available in medicine, biology, public health, environmental science and AI. Well-known monographs include Ramsay and Silverman (2005, 2007), Ferraty and Vieu (2006), and Zhang (2013). In this paper, we focus on the following varying-coefficient model with functional response:

$$y_i(s) = x_i^T \boldsymbol{\beta}(s) + \varepsilon_i(s),$$

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