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TESTING EXOGENEITY IN THE FUNCTIONAL LINEAR REGRESSION MODEL

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Abstract: We propose a novel test statistic for testing exogeneity in the functional linear regression model. In contrast to Hausman-type tests in finite dimensional linear regression setups, we show that a direct extension to the functional linear regression model is not possible. Instead, we propose a test statistic based on the sum of squared differences of projections of the two estimators used for testing the null hypothesis of exogeneity in the functional linear regression model. We derive asymptotic normality under the null and show consistency under general alternatives. Moreover, we establish bootstrap consistency results for residual-based bootstrap approaches. In simulations, we investigate the finite sample performance of the proposed exogeneity tests and illustrate the superiority of bootstrap-based approaches. In particular, the bootstrap-based results turn out to be much more robust with respect to the choice of the regularization parameter.

Key words and phrases: Asymptotic theory, bootstrap inference, endogeneity, Hausman test, instrumental variables, inverse problem.

1. Introduction

The construction of goodness-of-fit tests in functional regression models is much more complicated than e.g. in the multiple linear setting. This is particularly due to the fact that, in functional linear regression models, the L_2-distance of the slope function estimator to the true function has no proper limiting distribution. Under exogeneity, this was shown in Cardot, Mas and Sarda (2007) and Ruymgaart et al. (2011) for two slope function estimators in the classical functional linear regression model. It turns out that this lack of a proper limiting distribution also remains for other such estimators based on different model assumptions such as endogeneity. This phenomenon inherent to (infinite-dimensional) functional data setups is probably the main reasons why testing in general and goodness-of-fit testing in particular is less developed for functional regression models. Especially, desirable and seemingly natural counterparts of standard and well-established tests from the (finite-dimensional) multiple linear regression model are still missing in functional linear regression setups.

In functional data settings, existing goodness-of-fit tests are described in Müller and Stadtmüller (2005), who use a suitable scalar product to transform

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