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MAXIMUM CONDITIONAL ALPHA TEST FOR CONDITIONAL MULTI-FACTOR MODELS

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Abstract: In this paper, a novel test, called the maximum conditional alpha (MCA) test, which enhances the testing power for detecting alpha in linear multi-factor models, is proposed. This test is specifically designed for conditional multi-factor models with time-varying coefficients, where the number of test assets (N) exceeds the number of observations (T) and the alternative hypothesis is a sparse vector, meaning that only a few components violate the null hypothesis. By carefully studying the estimation error derived from the B-spline estimation, we rigorously demonstrate that the proposed test converges to a type-I extreme value distribution when $\min(T, N)$ tends to infinity, subject to mild conditions. Furthermore, the proposed MCA test was extended to incorporate latent factors within conditional multi-factor models. The small-sample properties of the proposed MCA test were assessed via Monte Carlo simulations. Finally, the proposed method was applied to evaluate the efficiency of the U.S. stock market using the conditional Fama-French three-factor model. The results demonstrate that the MCA test outperforms existing tests in terms of statistical power.

Key words and phrases: B-spline estimator, maximum conditional alpha test, sparse alternative, time-varying coefficient.

1. Introduction

Explaining the variations in average returns across different assets is a fundamental question in finance. The capital asset pricing model (CAPM), pioneered by Sharpe (1964) and Lintner (1965), has long been the cornerstone of asset pricing. However, CAPM has traditionally been the key framework for asset pricing. However, the CAPM has proven to be inadequate, leading to the development of alternative multi-factor models like the widely used three-factor model proposed by Fama and French (1993). Typically, each factor in these multi-factor models has significant economic meaning and pricing ability.

In these models, denoting the excess return of asset i at time t as R_{it} and the $d \times 1$ observable vector of common factors as $\mathbf{f}_t = (f_{1t}, \ldots, f_{dt})^\top \in \mathbb{R}^d$, the linear multi-factor model with N test assets takes the following form:

$$R_{it} = \alpha_i + \boldsymbol{\beta}_i^\top \mathbf{f}_t + \varepsilon_{it}, \tag{1.1}$$

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