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STATISTICAL INFERENCE FOR HEAVY-TAILED AND PARTIALLY NONSTATIONARY VECTOR ARMA MODELS

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Abstract: This paper studies the full rank least squares estimator (FLSE) and reduced rank least squares estimator (RLSE) of the heavy-tailed and partially nonstationary ARMA model with the tail index $\alpha \in (0, 2)$. It is shown that the rate of convergence of the FLSE related to the long-run parameters is n (sample size) and that related to the short-term parameters are $n^{1/\alpha}\tilde{L}(n)$ and n, respectively, when $\alpha \in (1, 2)$ and $\in (0, 1)$. Its limiting distribution is a stochastic integral in terms of two stable random processes when $\alpha \in (0, 2)$ for the long-run parameters and is a functional of some stable processes when $\alpha \in (1, 2)$ for the short-run parameters. Based on FLSE, we derive the asymptotic properties of the RLSE. The finite-sample properties of the estimation are examined through a simulation study and an application to three U.S. interest rate series is given.

Key words and phrases: ARMA models, cointegration, full rank LSE, heavy-tailed time series, reduced rank LSE.

1. Introduction

Consider the m-dimensional time series $\{Y_t\}$ generated by the ARMA model:

$$\mathbf{\Phi}(L)\mathbf{Y}_t = \mathbf{\Theta}(L)\boldsymbol{\varepsilon}_t, \tag{1.1}$$

where $\mathbf{\Phi}(z) = \mathbf{I}_m - \sum_{i=1}^{p} \mathbf{\Phi}_i z^i$ and $\mathbf{\Theta}(z) = \mathbf{I}_m - \sum_{i=1}^{q} \mathbf{\Theta}_i z^i$ are matrix polynomials in z of degrees p and q, L is the backshift operator, $\mathbf{\Phi}_i (1 \le i \le p)$ and $\mathbf{\Theta}_i (1 \le i \le q)$ are $m \times m$ matrices, $\mathbf{I}_m$ denotes the $m \times m$ identity matrix, and $\{\boldsymbol{\varepsilon}_t\}$ is a sequence of independent and identically distributed (i.i.d.) m-dimensional vector noises. When all the roots of $\det\{\mathbf{\Phi}(z)\} = 0$ lie outside the unit circle, model (1.1) is stationary, where $\det\{\}$ means the determinant of a matrix. In this case, model (1.1) has been extensively applied in many areas such as economics and finance and its modeling procedure was fully established in Tsay (2014).

When $\det\{\mathbf{\Phi}(z)\} = 0$ has $d < m$ unit roots and the remaining roots outside the unit circle, model (1.1) is called the partially nonstationary ARMA model.

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