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MEAN DIMENSION REDUCTION AND TESTING FOR NONPARAMETRIC TENSOR RESPONSE REGRESSION

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Abstract: In this article, we propose a flexible model-free approach to the regression analysis of a tensor response and a vector predictor. Without specifying the specific form of the regression mean function, we consider two closely related statistical problems: (i) estimation of the dimension reduction subspace that captures all the variations in the regression mean function, and (ii) hypothesis testing of whether the conditional expectation of a linear dimension reduction of the response given the predictor is invariant to the changes in the predictor. We propose a new nonparametric metric called tensor martingale difference divergence, and study its statistical properties. Built on this new metric, we develop computationally efficient estimation and asymptotically valid testing procedures. We demonstrate the efficacy of our method through both simulations and two real data applications for macroeconomics and e-commerce.

Key words and phrases: Martingale difference divergence, nonlinearity, sufficient dimension reduction, tensor decomposition, tensor regression, wild bootstrap.

1. Introduction

Tensor data is now becoming ubiquitous in a wide range of scientific and business applications. For instance, in economics, multiple macroeconomic indices at varying time points across different countries are assembled as a three-way tensor. In neuroscience, images obtained by anatomical or functional magnetic resonance imaging take the form of three-way or four-way tensors. Tensor data analysis is thriving in statistics and machine learning in recent years. See Bi et al. (2021), and Sun, Hao and Li (2021) for reviews.

A central question in tensor analysis is finding meaningful low-dimensional tensor structures given the complex and high-dimensional tensor data. There is a line of research modeling the tensor as predictor or response in a regression setting. Early tensor predictor regression solutions focus on parametric and usually linear or generalized linear type models (Zhou, Li and Zhu, 2013; Li et al., 2018; Zhang and Li, 2017; Chen, Raskutti and Yuan, 2019). More recently, Hao et al. (2021), and Zhou, Wong and He (2024) extended tensor predictor regression to nonparametric models through basis expansion. Relatedly,

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