Statistica Sinica **6**(1996), 631-648

A MULTI-STEP ANALYSIS PROCEDURE FOR ROBUST DESIGN

Kwok-Leung Tsui

Hong Kong University of Science and Technology

Abstract: Robust Design is an important method for improving product or manufacturing process design. In analyzing robust design experiments with a fixed target, Taguchi proposed a two-step procedure to identify the "optimal" factor settings that minimize the variance and adjust the mean to target. This paper generalizes the two-step procedure to a multi-step procedure based on the response model approach. We derive and illustrate the procedure under various conditions. We also show that the "optimal" solution from the procedure is invariant with respect to a general class of loss functions.

Key words and phrases: Taguchi's method, response model, loss model, product design, two-step procedure.

1. Introduction

In 1980, Taguchi (1986) introduced the robust design method to several major American industries, which resulted in significant quality improvement in product and manufacturing process design. In analyzing robust design experiments where the response of interest has a fixed target, Taguchi proposed a two-step procedure for identifying the "optimal" factor settings: (i) calculated the signal-to-noise (SN) ratios and find the control factor settings which maximize the SN ratio, (ii) shift mean response to the target by changing the adjustment factor.

As explained in Leon, Shoemaker and Kacker (1987) (hereafter called LSK), there are two major advantages of the two-step procedure: (i) it reduces the dimension of the original optimization problem; (ii) it does not require re-optimization for future changes of the target value. In terms of statistical modeling, Taguchi's approach is a special case of the following "loss model" approach: first compute estimates of loss measures (Taguchi proposed the SN ratio as the loss measure), then determine the "optimal" factor settings by fitting a model to these loss estimates. An alternative modeling approach is to first model the observed response, and then determine the "optimal" factor settings from the fitted response model. This approach is called the "response model" approach and was first proposed by Welch et al. (1990) as a formal procedure. Recognizing the advantages of the response model approach, Shoemaker, Tsui and Wu

(1991) (hereafter called STW) developed analysis techniques for examining individual control-by-noise (C×N) interaction plots to reveal control factor levels that dampen the effects of individual noise factors. Related approaches have also been discussed by Box and Jones (1990) and Lucas (1990). Although the response model approach used in the multi-step procedure has several advantages over the loss model approach (see STW (1991)) the former approach requires more work and depends more critically on the adequacy of the fitted model. A comparison of the response model and loss model approaches can be found in Tsui (1996).

This paper generalizes the two-step procedure to a multi-step procedure based on the noise decomposition idea discussed by STW (1991) and Shoemaker and Tsui (1993). This multi-step procedure extends the advantages of both the two-step procedure and the response model approach. Better than the two-setp procedure, the multi-step procedure further reduces the dimension of the original optimization problem and does not require re-optimization for future elimination of noise factors as well as changes of the target value. Moreover, in addition to helping the experimenter reveal control factor levels that dampen the effects of individual noise factors, the new procedure allows the experimenter to make tradeoffs among the off-target bias and the individual variances contributed by different noise factors.

In addition, we discover an *important* advantage of the two-step procedure that has never been addressed. We show that the "optimal" solution from the two-step (or multi-step) procedure is invariant to the choice of the average loss function within the class of loss functions proposed by Box and Jones (1990).

The rest of the paper is organized as follows. In Section 2 we define the robust design problem and objective and explain how we can fix the noise factor in the experiment to study the process mean and variance of the manufacturing process. In Section 3 we propose a multi-step procedure under a response model and characterize the classes of models where the procedure would work. Section 4 discusses practical situations when the variation caused by individual noise factors can not be completely eliminated due to physical and engineering constraints. Section 5 considers a general class of average loss functions proposed by Box and Jones (1990) and shows that the "optimal" control factor settings resulting from the multi-step procedure will remain "optimal" within the whole class of average loss functions. Section 6 illustrates the multi-step procedure with a real example. The paper is concluded in Section 7.

2. Fixed Noise and Random Noise

In this paper we concentrate on robust design problems where the response of interest has a known fixed target. (This type of response is called the on-target or nominal-the-best (NTB) quality characteristic.) We assume that the

experimenter has identified all the major control factors ($\boldsymbol{C}$) and "external" noise factors ($\boldsymbol{N}$) that are the sources of variaton in the manufacturing process. (As pointed out in the discussion by Shoemaker and Tsui in Nair (1992), the robust design method would be much less efficient if there are no "external" noise factors.) In general, we assume that the response of interest (Y) is affected by the control and noise factors as follows:

$$Y = f(\boldsymbol{C}, \boldsymbol{N}) + \epsilon, \tag{2.1}$$

where $\boldsymbol{C} = (C_1, \ldots, C_p)^T$, $\boldsymbol{N} = (N_1, \ldots, N_q)^T$, and f is a transfer function. The control factors $\boldsymbol{C}$ are assumed to be fixed since they are controllable during production. In the situation where there is a random shift of the nominal value of the control factor, an additional noise factor can be added to the model to represent the shift (see Welch et al. (1990) for an example). The noise factors $\boldsymbol{N}$ are assumed to be random and follow some distributions as the values of noise factors are not controllable during production. The pure error ϵ represents the remaining variability of the manufacturing process that is not captured by the noise factors. We assume that the dependence of ϵ on the control and noise factors is negligible, i.e., the ϵ's are independently and identically distributed with mean zero and variance σ^2 and are independent of the noise factors.

Similar to LSK (1987) and Leon and Wu (1992), we define the robust design objective as follows. For given values of control factors, the values of the response Y are determined by the values of noise factors $\boldsymbol{N}$ and ϵ through the transfer function f. A *loss* occurs if Y differs from a fixed target t that represents the ideal response. Since both $\boldsymbol{N}$ and ϵ are random, the average loss is $R(\boldsymbol{C}) = E_{\boldsymbol{N},\epsilon}(L(Y,t))$, where $L(Y,t)$ is a loss function. The robust design objective is to choose the values of the control factors to minimize the average loss. In practice, this minimization may be subject to a constraint such as the mean on target constraint, $E(Y) = t$. A common choice of loss function is the quadratic loss function popularized by Taguchi (1986), $L(Y,t) = A_0(Y-t)^2$, where A_0 is a constant. (See Leon and Wu (1992) for other classes of loss functions.) It follows that the average loss becomes:

$$\begin{aligned} R(\boldsymbol{C}) &= A_0 E_{\boldsymbol{N},\epsilon}(Y-t)^2 \\ &= A_0[\mathrm{Var}_{\boldsymbol{N},\epsilon}(Y) + (E_{\boldsymbol{N},\epsilon}(Y) - t)^2], \end{aligned} \tag{2.2}$$

where $\mathrm{Var}_{\boldsymbol{N},\epsilon}(Y)$ and $E_{\boldsymbol{N},\epsilon}(Y)$ are the mean and variance of the response of the manufacturing process over random noise $\boldsymbol{N}$ and ϵ, which we willl refer to as the process mean and process variance in the rest of the paper. Therefore, if the quadratic loss is believed to be a good approximation, the objective of robust design is to minimize the sum of the process variance and the square of the process bias (the bias of the process mean from the target).

In reality, the transfer function f is often unknown. To achieve the robust design objective, the designer needs to first run experiments to understand the relationship between the response and the control and noise factors, then determine the "optimal" control factor values. Although the noise factors are random during production, they are often treated as fixed during the experiment to increase the efficiency of the experiment (see the discussion by Shoemaker and Tsui in Nair (1992) for more explanation). In practice, after running the experiment, the experimenter would first fit an empirical model to either the response or the average loss, then determine the "optimal" control factor values based on this empirical model.

Since the experiment is usually done off-line, an implicit assumption behind this approach is that the fitted empirical model will continue to be a good approximation to the true transfer function f or the true average loss function during on-line production. This assumption is not always valid since it is actually an extrapolation of the fitted model. To increase the validity of the extrapolation assumption, the experimenter should simulate the manufacturing conditions as much as possible during the experiment.

Below we consider a simple approximation of the transfer function, an additive model with control-by-noise interactions, to illustrate how we can fix the noise factors in the experiment to estimate the process mean and variance during production. Suppose the response Y for fixed noise factors in the experiment can be expressed by the following model:

$$Y \mid \boldsymbol{N} = \alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{C} + \sum_{j=1}^{q} (\gamma_j + \boldsymbol{\beta}_j^T \boldsymbol{C}) N_j + \epsilon, \tag{2.3}$$

where $Y \mid \boldsymbol{N}$ is the conditional random variable Y given $\boldsymbol{N}$, $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_p)^T$ and $\boldsymbol{\beta}_j = (\beta_{j1}, \ldots, \beta_{jp})^T$. We assume no control-by-control interaction terms in (2.3). More general models including these terms will be addressed in Section 6.

As mentioned earlier, $\boldsymbol{N}$ is treated as fixed in the experiment although it is random during on-line production. It follows that the conditional mean and variance for fixed noise are $E(Y \mid \boldsymbol{N}) = \alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{C} + \sum_{j=1}^{q} (\gamma_j + \boldsymbol{\beta}_j^T \boldsymbol{C}) N_j$, and $\mathrm{Var}\,(Y \mid \boldsymbol{N})\sigma^2$.

Suppose the N_j's are independently distributed with mean zero and variance σ_{Nj}^2 during production. In order to use model (2.3) to estimate the process mean and variance, we apply the following conditional expectation relationships:

$$\begin{aligned} E_{\boldsymbol{N},\epsilon}(Y) &= E_{\boldsymbol{N}}[E(Y \mid \boldsymbol{N})] = \alpha_0 + \boldsymbol{\alpha}^T \boldsymbol{C}, & (2.4) \\ \mathrm{Var}_{\boldsymbol{N},\epsilon}(Y) &= \mathrm{Var}_{\boldsymbol{N}}[E(Y \mid \boldsymbol{N})] + E_{\boldsymbol{N}}[\mathrm{Var}\,(Y \mid \boldsymbol{N})] \\ &= \sum_{j=1}^{q} (\gamma_j + \boldsymbol{\beta}_j^T \boldsymbol{C})^2 \sigma_{Nj}^2 + \sigma^2. & (2.5) \end{aligned}$$

As mentioned above, formulae (2.4) and (2.5) are good approximations of the true process mean and process variance only if model (2.3) continues to be a good approximation of the true response model during on-line production where noise is random.

3. A Response Model Multi-Step Procedure

As explained in Section 2, the objective of robust design is to minimize the average loss, i.e., the sum of the process variance and the square of the process bias. According to (2.4), the process bias can be ideally minimized to zero by choosing appropriate values of the control factors such that $\alpha_0 + \sum_1^p \alpha_i C_i = t$. According to (2.5), the process variance can be ideally minimized to σ^2 by choosing appropriate values of the control factors such that $\gamma_j + \sum_1^p \beta_{ij} C_i = 0$ for all j. In general, in order to minimize the average loss, these two minimizations have to be done simultaneously since they involve the same set of control factors $C_1, \ldots, C_p$. However, if there exists a control factor (mean adjustment factor) such that the value of this factor does not affect the process variance and this factor can be used to shift the process mean to the target, the average loss can be minimized in two sequential steps, i.e., first minimize the process variance with respect to the other control factors, then shift the process mean to the target by changing the mean adjustment factor. This is called a two-step procedure and was discussed extensively in LSK (1987).

According to Shoemaker and Tsui (1993), the process variance in (2.5) can be further decomposed into q individual variance components contributed by the q noise factors, plus the pure error variance σ^2 . The following table illustrates the complete decomposition of the average loss:

Decomposition of average loss

Component	Coefficients					Mean or	Ideal	Squared Bias or
	C_1	C_2	$\cdots$	C_p	Intercept	$\partial Y/\partial N_j$	Value	Individual Variance
Mean	α_1	α_2	$\cdots$	α_p	α_0	$\sum \alpha_i C_i + \alpha_0$	t	$(\sum \alpha_i C_i + \alpha_0 - t)^2$
N_1	β_{11}	β_{21}	$\cdots$	β_{p1}	γ_1	$\sum \beta_{i1} C_i + \gamma_1$	0	$(\sum \beta_{i1} C_i + \gamma_1)^2 \sigma^2_{N_1}$
N_2	β_{12}	β_{22}	$\cdots$	β_{p2}	γ_2	$\sum \beta_{i2} C_i + \gamma_2$	0	$(\sum \beta_{i2} C_i + \gamma_2)^2 \sigma^2_{N_2}$
.	.	.	...	.	.	.	.	.
.	.	.	...	.	.	.	.	.
N_q	β_{1q}	β_{2q}	$\cdots$	β_{pq}	γ_q	$\sum \beta_{iq} C_i + \gamma_q$	0	$(\sum \beta_{iq} C_i + \gamma_q)^2 \sigma^2_{N_q}$
Pure Error								
Total								Average Loss

(3.1)

As shown above, instead of decomposing into only the squared bias and variance, the average loss can be decomposed into $(q+2)$ components: the squared

bias, the q individual process variances, and the pure error variance σ^2. As mentioned earlier, the process bias squared can be minimized to zero by choosing appropriate values of the C_i's to make the process mean equal to its ideal value t. Similarly, each of the individual process variances, $\sigma^2_{Y|N_j} = (\sum_1^p \beta_{ij}C_i + \gamma_j)^2\sigma^2_{Nj}$, can be ideally minimized to zero by choosing appropriate values of the $C_i's$ to make the gradient of Y with respect to N_j, $\partial Y/\partial N_j$, equal to zero.

This is an ideal situation for robust design since $\partial Y/\partial N_j$ is a measure of the sensitivity of Y with respect to a change in the noise factor N_j. When this sensitivity is reduced to zero, the response Y will be completely insensitive (or robust) to the noise factor N_j. The last component, pure error, is independent of the control factors, so its variance remains constant as σ^2 for all values of the C_i's. Therefore, ideally, the process bias and the q individual process variances can be minimized to zero by solving the following $q+1$ simultaneous linear equations of the p control factors, if a solution exists.

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_p \\ \beta_{11} & \beta_{21} & \cdots & \beta_{p1} \\ \beta_{12} & \beta_{22} & \cdots & \beta_{p2} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \beta_{1q} & \beta_{2q} & \cdots & \beta_{pq} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \cdot \\ \cdot \\ C_p \end{pmatrix} + \begin{pmatrix} \alpha_0 - t \\ \gamma_1 \\ \gamma_2 \\ \cdot \\ \cdot \\ \gamma_q \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{pmatrix}. \tag{3.2}$$

In terms of matrix notation, these equations can be written as $\mathbf{AC} + \mathbf{b} = \mathbf{0}$, where $\mathbf{A} = \begin{pmatrix} \boldsymbol{\alpha}^T \\ \mathbf{B} \end{pmatrix}$ with $\mathbf{B} = \{\beta_{ij}\}$, $\mathbf{b} = (\alpha_0 - t, \gamma_1, \ldots, \gamma_q)^T$, and $\mathbf{0} = (0, 0, \ldots, 0)^T$.

Note that a solution of (3.2) does not always exist in practice. When this is the case, the average loss can be minimized by a more general multi-step procedure described in the next section.

When a solution of (3.2) exists, the average loss can be minimized by the values of $C_1, \ldots, C_p$ that solve these $q+1$ simultaneous equations. In particular, if there are some special structures in these equations, the minimization of the average loss can be partitioned into several smaller optimization problems. For the two-step procedure, the special structure is the existence of a mean adjustment factor. In terms of the coefficients in (3.1) or (3.2), the definition of the mean adjustment factor is the control factor C_{i^*} such that $\alpha_{i^*} \neq 0$ and $\beta_{i^*j} = 0$ for all j.

Following the definition of the mean adjustment factor, we can define the individual variance tuning factor for N_{j^*} as the control factor C_{i^*} such that $\beta_{i^*j^*} \neq 0$, $\alpha_{i^*} = 0$, and $\beta_{i^*j} = 0$ for all $j \neq j^*$. Thus the individual variance tuning factor of N_{j^*} can be used to tune down the individual process variance

contributed by N_{j^*} without affecting the other individual process variances and the process mean.

Similar to the mean adjustment factor, the existence of the individual variance tuning factors will allow further partitioning of the original optimization problem. As shown later, if there exist a mean adjustment factor and an individual variance tuning factor for each noise factor, the optimization problem can be partitioned into $q+1$ subproblems and can be solved sequentially by a $(q+1)$-step procedure.

In general, the existence of mean adjustment factor and individual variance tuning factor depends on the structure of the matrix $\mathbf{A}$ defined after Equation (3.2). Below we will classify the special structure of matrix $\mathbf{A}$ and develop a corresponding multi-step procedure for each class of the special structure. For convenience of discussion, we first assume an ideal situation that $p \geq q+1$ and the ranges of all the control factors are in the real line (i.e., $\boldsymbol{C} \in \Re^n$) so that the system of Equations (3.2) will always have solutions. The development of a multi-step procedure under more practical situations will be discussed in the next section.

I. In the first class of models, we assume that the matrix $\mathbf{A}$ in (3.2) is a blocked diagonal matrix with $q_k \times p_k$ block matrix A_k, for $k = 1, \ldots, K, \sum_1^K p_k = p, \sum_1^K q_k = q+1$, and $p_k \geq q_k$ (the case when $p_k < q_k$ will be discussed later). Clearly, the problem of solving the system of Equations (3.2) can be decomposed into K subsets of systems of q_k equations, $\mathbf{A}_k\mathbf{C}_k + \mathbf{b}_k = \mathbf{0}$, where $\mathbf{C}_k$ and $\mathbf{b}_k$ are respectively the vectors of the p_k control factors and the q_k intercepts of $\mathbf{b}$ corresponding to the q_k equations. It follows that the average loss can be minimized by sequentially solving these K subsets of systems of equations, which can be considered as a K-step procedure analogous to the two-step procedure. An extreme case of this class of models is that $q_k = 1$ for all k. This implies that $K = q+1$ and the average loss can be minimized by a $(q+1)$-step procedure as discussed earlier. Note that it does not matter in what order we perform the K-step procedure because of the special structure of (3.3). However, the order of performing the steps does matter in the next class of models.

II. In the second class of models, we assume that the matrix $\mathbf{A}$ in (3.2) is a blocked triangular matrix, i.e.,

$$\mathbf{A} = \begin{pmatrix} A_{11} & 0 & 0 & \cdots & 0 \\ A_{12} & A_{22} & 0 & \cdots & 0 \\ A_{13} & A_{23} & A_{33} & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ A_{1K} & A_{2K} & A_{3K} & \cdots & A_{KK} \end{pmatrix}, \tag{3.3}$$

where the dimension of each block matrix A_{kk} is $q_k \times p_k$ with $p_k \geq q_k$, the dimension of each block matrix A_{lk} is $q_k \times p_l$ with $l < k$, $\sum_1^K p_k = p$, and

$\sum_1^K q_k = q + 1$. The problem of solving the system of Equations (3.2) can be decomposed to K subsets of systems of q_k equations, $\sum_{l=1}^{k} \mathbf{A}_{lk}\mathbf{C}_l + \mathbf{b}_k = \mathbf{0}$, where $\mathbf{C}_l$ and b_k are respectively the vectors of the p_l control factors and the q_k intercepts of $\mathbf{b}$ corresponding to the q_k equations. It follows that the average loss can be minimized by the following iterative procedure:
1. Find $\mathbf{C}_1^*$ that solves the system of equations $\mathbf{A}_{11}\mathbf{C}_l + \mathbf{b}_1 = 0$.
2. For $k = 2$ to K, find $\mathbf{C}_k^*$ that solves the system of equations :

$$\mathbf{A}_{kk}\mathbf{C}_k + \sum_{l=1}^{k-1} \mathbf{A}_{lk}\mathbf{C}_l^* + \mathbf{b}_k = 0.$$

It follows that the solution $\mathbf{C}^* = (\mathbf{C}_1^*, \ldots, \mathbf{C}_K^*)^T$ is the solution of the system of Equations (3.2). This iterative procedure is equivalent to a K-step procedure. The difference between this K-step procedure and the K-step procedure in Class I is that the K steps here have to be performed in the special order from $k = 1$ to K. Similarly, an extreme case of this class of models is that $q_k = 1$ for all k. This implies that $K = q + 1$ and the average loss can be minimized by a $(q + 1)$-step procedure following the special order from $k = 1$ to $q + 1$.

As an example, consider the two-step procedure for the additive model described in LSK (1987). It follows that in LSK's additive model, $K = 2$, $q_1 = q$, $q_2 = 1$, $\mathbf{C}_1$ equals their $\mathbf{d}$ vector, and $\mathbf{C}_2$ equals their $\mathbf{a}$ vector. We further assume that the control factors can take any possible values so that it is possible to reduce the process bias to zero and the process variance to σ^2. (The more general case where the process variance cannot be reduced to σ^2 will be discussed in next section.) Thus, the average loss is minimized by the two-step procedure: first finding the solution $\mathbf{C}_1^*$ of the system of equations $\mathbf{B}\mathbf{C}_1 + \gamma = \mathbf{0}$; then at $\mathbf{C}_1 = \mathbf{C}_1^*$, find the solution $\mathbf{C}_2^*$ of the equation $\boldsymbol{\alpha}^T\mathbf{C}_2 + \mathbf{B}\mathbf{C}_1^* + \alpha_0 = t$.

III. In this class there is no special structure in the matrix $\mathbf{A}$. The values of $\boldsymbol{C}$ that minimize the average loss can be obtained by solving the system of Equations (3.2) by any numerical method such as the simplex method. There are always solutions that minimize the average loss to σ^2 since it is assumed that $p \geq q + 1$ and $\boldsymbol{C} \in \Re^n$. The solution will be unique when $p = q + 1$ and the system of equations has full rank.

Note that the three classes of models have a hierarchical relationship, i.e., Class I is a special case of Class II, which is a special case of Class III. Therefore, each subproblem in the partition above can be considered as Class III problem. As illustrated in Section 5, the existence of Classes I and II are quite possible in practice. This agrees with the sparsity assumption given in Box, Hunter and Hunter (1978) and Taguchi (1986), which is quite common in industrial experiments.

4. A Multi-Step Procedure under Realistic Constraints

As mentioned in the last section, the average loss equals the sum of the process variance and the process bias squared. Ideally, the process bias squared can be reduced to zero by shifting the process mean to the target t, and the process variance can be reduced to σ^2 by making each individual process variance ($\sigma^2_{Y|N_j}$) equal zero. In practice, although it may be possible to shift the mean onto the target or to make some $\sigma^2_{Y|N_j}$, equal zero, it is unlikely that we would be able to make all of them equal zero because of the following realistic constraints:
1. Under physical or engineering constraints, the feasible ranges of the control factor are limited, i.e., $L_i \le C_i \le R_i$ for all i.
2. Some control factors may be categorical (or qualitative) so that the only possible values of these factors are those we tested in the experiment.
3. There are more noise factors than control factors, i.e., $p < q + 1$.

In reality, the robust design problem can be under one or several of the constraints listed above. Welch et al. (1991), STW (1991), Shoemaker and Tsui (1993), and many other papers consider the problem under the second constraint, i.e., the only possible values of the control factors are those values tested in the experiment. Kim and Myers (1992) consider the problem under the third constraint, i.e., $p < q+1$ and $\boldsymbol{C} \in \Re^n$. They apply response surface methodology techniques to minimize the process variance. The first constraint, although very common in practice, has seldom been discussed in the context of robust design problems. Typically, a robust design problem often contains both continuous and categorical control factors where the continuous factors are under some physical constraints on their feasible values. Note that the first constraint above is quite common in practice but does not cover general constraints that involve several control factors simultaneously. These general constraints will be addressed in the Discussion.

When it is impossible to make the squared bias and all individual process variances equal zero because of the constraints described above, there will be no solutions for the system of Equations (3.2). Recall that the original robust design objective is to minimize the average loss defined in (2.2)− a weighted sum of squares of the bias and the gradients $\partial Y/\partial N_j$'s with weights 1 for the bias and $\sigma^2_{N_j}$ for the gradients, respectively. In matrix notation, the general robust design problem under (2.3) is defined as:

$$\text{Minimize } R(\boldsymbol{C}) = (\mathbf{A}\boldsymbol{C} + \mathbf{b})^T \mathbf{D} (\mathbf{A}\boldsymbol{C} + \mathbf{b}) + \sigma^2 \text{ with respect to } \boldsymbol{C} \in \Im, \quad (4.1)$$

where $\mathbf{D}$ is a diagonal matrix with diagonal elements $d_{11} = 1$, $d_{jj} = \sigma^2_{N_{j-1}}$ for $j = 2, \ldots, (q+1)$, and $\Im$ is the set of feasible values of the control factor under some realistic constraints.

It is possible to decompose the general optimization problem (4.1) into subproblems and develop a multi-step procedure. The general partition of optimization problems has been studied in the past and a good review can be found in Duff (1977). The methods proposed in this paper take advantage of the simple structure assumed in the models and provide more information on understanding the manufacturing process. For Class I models described in Section 3, it can be easily seen that the general optimization problem (4.1) can be decomposed into the following K sub-optimization problems:

$$\text{Minimize } (\mathbf{A}_k\mathbf{C}_k + \mathbf{b}_k)^T\mathbf{D}_k(\mathbf{A}\mathbf{C}_k + \mathbf{b}_k) \text{ with respect to } \mathbf{C}_k \in \Im, k = 1, \ldots, K,$$

where $\mathbf{D}_k$ is the kth diagonal block of the matrix $\mathbf{D}$. This is a trivial situation. It is interesting to note that, when $K = q + 1$, the solutions of the general problem (4.1) do not depend on the values of $\sigma^2_{N_{j-1}}$ although these terms appear in the objective function.

For Class II models described in Section 3, the optimization problem (4.1) cannot always be decomposed into K subproblems. As shown in the Appendix A of Tsui (1993), any average loss function $R(\mathbf{x}_1, \ldots, \mathbf{x}_M)$ can be minimized by the following general multi-step procedure.

1. Find $\mathbf{x}_1^*$ that minimizes $P_1(\mathbf{x}_1)$.
2. For $m = 2$ to M, find $\mathbf{x}_m^*$ that minimizes $P_m(\mathbf{x}_1^*, \ldots, \mathbf{x}_{m-1^*}, \mathbf{x}_m)$,

where P_m is the mth intermediate performance measure and defined to be

$$P_m(\mathbf{x}_1, \ldots, \mathbf{x}_m) = \min_{\mathbf{x}_{m+1}, \ldots, \mathbf{x}_M} R(\mathbf{x}_1, \ldots, \mathbf{x}_M), \text{ for } m = 1, \ldots, M.$$

This general multi-step procedure is an extension of the general two-step procedure described in LSK (1987). They call P_1 the performance measure independent of adjustment. As pointed out in the paper, even though this procedure is always possible, it is not always useful since the intermediate performance measures P_m's are sometimes very hard to derive. However, under model (2.3) and the special Class II structure of $\mathbf{A}$ in (3.3), the intermediate performance measures can be derived as follows. (See Leon and Wu (1992) for similar derivations under other conditions.)

It follows from (3.5) that the average loss equals:

$$\begin{aligned} R(\boldsymbol{C}) &= (\mathbf{A}_{11}\mathbf{C}_1 + \mathbf{b}_1)^T\mathbf{D}_1(\mathbf{A}_{11}\mathbf{C}_1 + \mathbf{b}_1) + \cdots \\ &\quad + \Big(\sum_{l=1}^{K} \mathbf{A}_{lK}\mathbf{C}_l + \mathbf{b}_K\Big)^T\mathbf{D}_K\Big(\sum_{l=1}^{K} \mathbf{A}_{lK}\mathbf{C}_l + \mathbf{b}_K\Big) + \sigma^2 \\ &= \sum_{k=1}^{K} R_k(\mathbf{C}_1, \ldots, \mathbf{C}_k) + \sigma^2, \end{aligned}$$

where $R_k(\mathbf{C}_1, \ldots, \mathbf{C}_k) = (\sum_{l=1}^{k} \mathbf{A}_{lk}\mathbf{C}_1 + \mathbf{b}_k)^T \mathbf{D}_k (\sum_{l=1}^{k} \mathbf{A}_{lk}\mathbf{C}_l + \mathbf{b}_k)$ for $k = 1, \ldots, K$.

Suppose for any given values of $\mathbf{C}_1, \ldots, \mathbf{C}_{k-1} \in \Im$, say $\mathbf{C}_1^{**}, \ldots, \mathbf{C}_{k-1}^{**}$, there exists $\mathbf{C}_k^{**} \in \Im$ such that

$$R_k(\mathbf{C}_1^{**}, \ldots, \mathbf{C}_k^{**}) = \min_{\mathbf{C}_1, \ldots, \mathbf{C}_k \in \Im} R_k(\mathbf{C}_1, \ldots, \mathbf{C}_k) = R_{k0} \text{ for } k = 2, \ldots, K, \quad (4.2)$$

where R_{k0} is the absolute minimum of $R_k(\mathbf{C}_1, \ldots, \mathbf{C}_k)$ for $\boldsymbol{C} \in \Im$. It follows that, for $k = 1, \ldots, K$,

$$\begin{aligned} P_k(\mathbf{C}_1, \ldots, \mathbf{C}_k) &= \min_{\mathbf{C}_{k+1}, \ldots, \mathbf{C}_K} R(\mathbf{C}_1, \ldots, \mathbf{C}_K) \\ &= \sum_{l=1}^{k} R_l(\mathbf{C}_1, \ldots, \mathbf{C}_l) + \sum_{l=k+1}^{K} \min_{\mathbf{C}_{k+1}, \ldots, \mathbf{C}_K} R_l(\mathbf{C}_1, \ldots, \mathbf{C}_l) \\ &= \sum_{l=1}^{k} R_l(\mathbf{C}_1, \ldots, \mathbf{C}_l) + \sum_{l=k+1}^{K} R_{l0}, \end{aligned} \quad (4.3)$$

since the terms in the second summation of (4.3) all reduce to their absolute minimum for any given values of $\mathbf{C}_1, \ldots \mathbf{C}_k$.

Since there exist intermediate performance measures for model (2.3), we can apply the general multi-step procedure to minimize the average loss in (4.1). Therefore, under (2.3) and (3.3), (4.1) can be minimized by the following multi-step procedure:

1. Find $\mathbf{C}_1^* \in \Im$ that minimizes $R_1(\mathbf{C}_1)$. (4.4)
2. For $k = 2$ to K, find $\mathbf{C}_k^* \in \Im$ that minimizes $R_k(\mathbf{C}_1^*, \ldots, \mathbf{C}_{k-1}^*, \mathbf{C}_k)$.

Note that minimizing $R_k(\mathbf{C}_1^*, \ldots, \mathbf{C}_{k-1}^*, \mathbf{C}_k)$ in steps 2 to K is equivalent to minimizing $P_k(\mathbf{C}_1^*, \ldots, \mathbf{C}_{k-1}^*, \mathbf{C}_k)$ since the other terms of P_k in (4.3) all reduce to constants for given $\mathbf{C}_1^*, \ldots, \mathbf{C}_{k-1}^*$.

An obvious special case of condition (4.2) to be satisfied is that $\Im = \Re^n$, i.e., $\boldsymbol{C} \in \Re^n$, which is the case in Section 3. Since $R_k(\mathbf{C}_1, \ldots, \mathbf{C}_k)$ can always be minimized to zero for any fixed values of $\mathbf{C}_1, \ldots, \mathbf{C}_{k-1}$ and zero is the absolute minimum of R_k, condition (4.2) is satisfied.

As pointed out in Wu (1987) and Leon and Wu (1992), condition (4.2) is only a sufficient condition for the multi-step procedure (4.4) to work. A weaker condition for the procedure to work is that (4.2) is true only when the first $k-1$ factors are at their optimal values from the earlier steps, i.e., for given values of $\mathbf{C}_1^*, \ldots, \mathbf{C}_{k-1}^*$, there exists $\mathbf{C}_k^* \in \Im$ such that

$$R_k(\mathbf{C}_1^*, \ldots, \mathbf{C}_k^*) = \min_{\mathbf{C}_1, \ldots, \mathbf{C}_k \in \Im} R_k(\mathbf{C}_1, \ldots, \mathbf{C}_k) = R_{k0} \text{ for } k = 2, \ldots, K.$$

A common situation for procedure (4.4) to work is that, at the kth step, there exist values of $\mathbf{C}_k \in \Im$ for $k = 2, \ldots, K$ suh that the bias or any gradient in R_k can be reduced to zero at the optimal values of the earlier steps, i.e., there exist valus of $\mathbf{C}_k$ that solve the subsystem of equations, $\mathbf{A}_{kk}\mathbf{C}_k + \sum_{l=1}^{k-1} \mathbf{A}_{lk}\mathbf{C}_l^* + \mathbf{b}_k = \mathbf{0}$. (In this case, minimizing R_k is equivalent to solving the subsystem of equations since there exists a solution.) This situation will be illustrated with a real example in the next section.

The two-step procedure for the additive model described in LSK (1987) is a special case of procedure (4.4). As described in their paper, the condition for the two-step procedure to work is the existence of an adjustment factor, a factor which is independent of the process variance and can be used to shift the mean to the target at the optimal values of the other factors. This condition is equivalent to the condition that $\mathbf{A}$ belongs to Class II models and the subsystem equation (the single equation of the process bias) has a solution at the optimal values of the other factors.

Note that since the average loss function (4.1) is a quadratic function of the control factors, it can be minimized by directly setting its first derivative to zero and solving a linear systems of p equations. However, this approach does not result in a decomposition of the average loss to process bias squared and individual process variances, and thus cannot be used to derive the multi-step procedure.

As mentioned before, the multi-step procedure extends the advantages of both the two-step procedure and the response model approach. Since the original optimization problem can be partitioned into several subproblems, the dimension of the original problem has been further reduced. Also, if it is decided in the future to eliminate some of the noise factors to satisfy the manufacturing specifications, re-optimization wil not be required if these noise factors are the components in the subproblems. Moreover, the decomposition provides a very clear picture for the experimenter to make tradeoffs on which component of the individual process variances and the bias to eliminate. This will be illustrated by the example in Section 6. Finally, as mentioned before, the "optimal" solution from the multi-step procedure remains "optimal" for a wide range of average loss functions. This advantage is discussed in detail in the next section.

5. An Invariance Property of the Multi-Step Procedure

The multi-step (or two-step) procedure has a very *important* property that has never been addressed. The property is that, when the conditions are met, the "optimal" solution from the multi-step (or two-step) procedure that minimizes the quadratic loss (2.2) will remain "optimal" within the general class of loss functions introduced by Box and Jones (1990).

Box and Jones (1990) considered the following average loss function as an overall measure of robust performance (which we refer to as the weighted mean variance loss in the rest of the paper):

$$R_\lambda(\boldsymbol{C}) = A_0[\lambda(E(Y) - t)^2 + (1 - \lambda)\mathrm{Var}\,(Y)],\ 0 \le \lambda \le 1. \tag{5.1}$$

This class of average loss functions does not correspond to any loss functions for an individual response except when $\lambda = 0.5$. As argued in Nair and Pregibon (1988), it is more reasonable to assume a quadratic model for the average loss function than for the loss of an individual response. We agree with the approach taken in Box and Jones (1990) and Nair and Pregibon (1988). We prefer to consider the average loss function as a weighted sum of the mean and variance of the distribution of the response.

The average loss function (5.1) is motivated by the fact that the average quadratic loss function (2.2) contains an arbitrary element and puts equal weights on the bias squared and variance. Note that (2.2) is a special case of (5.1) with $\lambda = 0.5$. As argued in Box and Jones (1990), "Unfortunately, the relative importance placed upon [the mean and variance] if we use the portmanteau criterion [the loss in (2.2)] is entirely dependent on how we choose t. If we give a higher value to t then more emphasis will be given to [the bias squared] and less to [the variance], and *vice versa*." The value of t may be different for different product designs, which creates an arbitrary element in the problem. In addition, there is no reason to have the same weight for the bias squared and variance as in (2.2) for different values of t. On the contrary, the average loss (5.1) will give the experimenter flexibility to weight the bias squared and variance for different design problems based on the engineering knowledge.

Although the average loss (5.1) gives more flexibility to the experimenter, a different choice of weight (λ) may require different analysis methods and result in different "optimal" solution in general. In practice, it may be hard for the experimenter to know what weight to choose for some problems and misspecification of weight may result in a "non-optimal" solution. However, as shown in Appendix B of Tsui (1993), if the conditions are met so that the multi-step procedures can be used to minimize the quadratic loss (2.2),the "optimal" solution from the procedure also minimizes the average loss function of (5.1) for any value of λ.

This invariance property is a very important advantage of the multi-step (two-step) procedure. When the conditions of the procedure are met, the experimenter does not need to know the exact weight of the average loss function. The solution from the procedure is always "optimal" for any choice of the average loss within the class of weighted mean variance loss functions. On the other hand, if the multi-step procedure cannot be applied to solve the robust design problem,

the "optimal" solution that minimizes the average loss (5.1) will be different for different choices of λ.

6. A Power Supply Transformer Example

This experiment addressed a transformer processing problem involving inductance changes. Due to inductance falling out of the required ranges, much time and cost had been incurred in dealing with the problem. The primary objective of the experiment was to minimize the variation of inductance from transformer to transformer after processing and to maintain the process mean to the target $t = 9.75$. Since the distribution of the noise was not specified in the original paper, we assume that the components are independent of each other and all have mean zero and variance 1. The process was described in Pfaff (1987).

Table 5.1. Design and data of the transformer example

Run	Control Array									Noise Array			
									r	-1	-1	1	1
									s	-1	1	-1	1
									t	-1	1	1	-1
	L	A	B	C	D	E	F	G	H	Data			
1	1	-1	1	1	-1	1	-1	-1	1	9.44	10.21	9.54	9.73
2	2	-1	1	1	-1	-1	1	1	-1	9.07	9.68	8.82	8.84
3	3	-1	1	-1	1	1	-1	1	-1	8.41	7.23	8.87	8.17
4	4	-1	1	-1	1	-1	1	-1	1	10.20	10.48	10.62	11.08
5	1	1	-1	-1	1	-1	1	1	-1	9.56	8.39	8.85	7.87
6	2	1	-1	-1	1	1	-1	-1	1	9.08	9.18	9.30	8.94
7	3	1	-1	1	-1	-1	1	-1	1	9.30	8.11	9.43	9.04
8	4	1	-1	1	-1	1	-1	1	-1	9.72	9.83	9.92	9.85
9	5	-1	-1	1	1	1	1	-1	-1	9.10	8.88	9.43	10.08
10	6	-1	-1	1	1	-1	-1	1	1	9.63	9.77	9.90	9.73
11	7	-1	-1	-1	-1	1	1	1	1	9.94	9.17	10.40	9.15
12	8	-1	-1	-1	-1	-1	-1	-1	-1	9.63	7.85	9.52	7.87
13	5	1	1	-1	-1	-1	-1	1	1	10.11	8.52	9.84	7.83
14	6	1	1	-1	-1	1	1	-1	-1	9.89	10.65	10.19	10.71
15	7	1	1	1	1	-1	-1	-1	-1	10.20	9.87	10.87	10.73
16	8	1	1	1	1	1	1	1	1	8.72	8.94	9.14	8.91

There are nine control factors, A, B, C, D, E, F, G, H, and L in the experiment. Each factor was tested at two levels except for factor L for which eight levels were tested. The control array was constructed by customizing a 2^{15-11} saturated fractional factorial design. Seven columns of the saturated design were

collapsed to generate an eight-level column for factor L and the remaining eight factors were assigned to the other eight columns. For the noise array, three two-level noise factors, r, s, and t, were assigned to a 2^{3-1} saturated fractional factorial design. As shown in Table 5.1, the resulting experiment plan is a product array of two saturated designs for control factors and noise factors.

Following the response model approach, we first modeled the response over both the control and noise factors. We decomposed factor L into seven orthogonal contrasts $(L_1, \ldots, L_7)$ which correspond to the seven columns of the original 2^{15-11} saturated design. The following fitted model was obtained using the method in Lenth (1989) to identify significant effects at the 0.10 significance level under the assumption that main effects are more important than 2-factor interactions.

$$\begin{aligned}\hat{y} \;=\; & 9.41 + .14B + .11C - .26G + .13L_1 - .11L_3 + .15L_4 - .24L_5 - .47L_7 \\ & + .18s + .12L_2r + .10Bs + .18Cs + .13Es - .11Gs + .15L_{4^s} - .13L_{7^s}. \quad (6.1)\end{aligned}$$

Based on the fitted model above, the decomposition table discussed in Section 3 can be constructed as follows:

Decomposition table of the transformer example

Comp.	B	C	G	L_1	L_3	L_4	L_7	L_2	E	L_5	Inter.	Bias or $\partial Y/\partial N_j$
r	0	0	0	0	0	0	0	.12	0	0	0	$\sum \beta_{i1}C_i + \gamma_1$
s	.1	.18	−.11	0	0	.15	.13	0	.13	0	−.18	$\sum \beta_{i2}C_i + \gamma_2$
Mean	.14	.11	−.26	.13	−.11	.15	−.47	0	0	−.24	−.34	$\sum \alpha_i C_i + \alpha_0 - t$

To identify the "optimal" control factor values, we first assume that the values of all control factors are continuous and can take any values on the real line. In addition, for purposes of illustration, we hypothetically assume that the contrasts L, $L_1, \ldots, L_7$ are actual control factors.

The table above illustrates a very clear picture of how each control factor affects the process bias and the individual process variance. It is found that control factors $B, C, G, L_1, L_3, L_4, L_5$, and L_7 affect the process bias, only L_2 affects the individual process variance caused by noise factor r, factors B, C, E, G, L_4, and L_7 affect the individual process variance caused by noise factor s, and the noise factor t does not contribute any variation to the process variance. This table allows the experimenter to identify the mean adjustment factor and individual variance tuning factors. Clearly L_1, L_3, and L_5 can be classified as mean-adjustment factors since these three control factors affect the mean but do not affect any of the individual process variances. Similarly, L_2 and E are the individual variance tuning factors for noise factors r and s, respectively.

According to the classifications in Section 3, the structure of the decomposition table above belongs to Class II, with $q_k = 1$ for all k. We follow the algorithm described in Section 3 to identify the control factor values that minimize the average loss. First, we chose $L_2^* = 0$ to reduce the individual variance $\sigma^2_{Y|r}$ to zero. Second, to reduce the individual variance $\sigma^2_{Y|s}$ to zero, we could have chosen any arbitrary values for $B, C, G, L_1, L_3, L_4, L_7$. We chose their values as $B^* = -1$, $C^* = 1$, $G^* = 1$,$L_1^* = 1$, $L_3^* = -1$, $L_4^* = 1$, $L_7^* = -1$, which are the best testing values for the discrete case as shown later. Then we determined the value of E by solving the second linear equation, $E^* = (.1B^* + .18C^* - .11G^* + .15L_4^* + .13L_7^* - .18)/.13 = -1.46$. Finally, to reduce the bias to zero, we determined the value of L_5 by solving the third linear equation, $L_5^* = (.14B^* + .11C^* - .26G^* + .13L_1^* - .11L_3^* + .15L_4^* - .47L_7^* - .34)/(-.24) = -0.96$. This choice of control factor values reduces the bias and the individual variance of each noise factor to zero so that the total average loss in minimized to σ^2.

Note that the choice of E^* (-1.46) is outside the experimental range $(-1, 1)$. If this range is a constraint to the experimenter as described in Section 4, the values of $B, C, G, L_1, L_3, L_4, L_7$ can be changed so that the choice of E is within $(-1, 1)$ but still reduce $\sigma^2_{Y|s}$ to zero. One such choice is $E^* = -1$, $L_5^* = -.06$, $L_7^* = -.54$ and all other control factors remain unchanged.

Suppose that instead of taking continuous values, the possible values of all control factors are restricted to those values tested in the experiment. In this case, the multi-step procedure cannot be used to minimize the average loss since the condition described in Section 4 is not satisfied. However, the decomposition table described in Section 3 can still be used to help the experimenter make tradeoffs among the off-target bias and the individual process variances contributed by different noise factors. This was illustrated in Tsui (1993).

7. Discussion

By taking advantage of the special structure of the model of the response, the minimization of the average loss can be decomposed into several sub-optimization problems. This decomposition leads to a multi-step procedure, which extends the advantages of both Taguchi's two-step procedure and the response model approach discussed in STW (1991). The new procedure further reduces the dimension of the original optimization problem and does not require re-optimization for future eliminations of noise factors as well as changes of the target value. The procedure also allows the experimenter to make tradeoffs among the off-target bias and individual process variances contributed by different noise factors. Moreover, the "optimal" solution from the two-step (or multi-step) procedure remains "optimal" within the class of loss functions proposed by Box and Jones (1990).

Another advantage of this approach is that it can be applied to smaller-the-better (STB) and larger-the-better (LTB) robust design problems. Contrary to the concept that there is no two-step procedure for these problems, as shown in Tsui and Li (1994), it is possible to develop a multi-step procedure for these problems. In addition, it is possible to apply the approach to problems with multiple responses which have different goals (NTB, STB, LTB etc.). However, in these problems the systems of equations will contain more equations and it is unlikely to be able to decompose the problems into multiple steps. Thus the multi-step procedure proposed in this paper may not be appropriate for problems with multiple responses.

An alternative to these problems, as suggested by a referee, is to determine the space of control factor values that solve each sub-optimization problem for each response, then explore all these spaces of values together to find a common solution. This approach increases the chance of finding a solution and is more appropriate for multi-response problems. Also, as pointed out in Section 4, the proposed multi-step procedure does not provide solutions for the problems with constraints that involve several control factors simulatneously. Instead, this alternative approach may be used to handle this type of constraints since a larger solution space is available. It is of great interest to further investigate this approach and to develop practical strategies and test them with real examples.

In this paper we concentrate on the additive model described in LSK (1987). For the multi-plicative model or other models in Box (1988), the transformation approach proposed there can be applied so that the model on the transformed response is additive. However, the criterion for identifying the appropriate transformation will be different since the response model approach is used instead. It will be interesting to develop new graphical methods to determine the appropriate transformation. Once the transformation is identified, the method proposed in this paper can be applied to the transformed response to make the process robust against the noise factors.

Finally, the approach in this paper is applicable as long as the general transfer function f can be approximated by an additive model with control-by-noise interactions. Note that the gradient of Y with respect to the noise, which is represented by the control-by-noise interactions, is the key for robustness improvement. As shown in Tsui (1993), if the transfer function f needs to be approximated by a full quadratic model, the approach is still applicable except that the process bias equation becomes quadratic.

Acknowledgement

The author would like to thank the associate editor, the referees, D. Goldsman, and A. C. Shoemaker for their helpful comments. This work was supported

by National Science Foundation Grants #DDM-9257918 and #DDM-9312698, the IBM TQM/CQI Research Grant, and the Alcoa Foundation Grant.

References

Box, G. E. P. (1988). Signal-to-noise ratios, performance criteria, and transformations. *Technometrics* **30**, 1-17.

Box, G. E. P., Hunter, W. G. and Hunter, J. S. (1978). *Statistics for Experimenter*. John Wiley, New York.

Box, G. E. P. and Jones, S. (1990). Designs for minimizing the effects of environmental variables. Technical Report, University of Wisconsin.

Duff, I. S. (1997). A survey of sparse matrix research. *Proceedings of the IEEE* **65**, 500-535.

Kim, Y. G. and Myers, R. H. (1992). A response surface approach to data analysis in robust parameter design. Technical Report 92-12, Virginia Polytechnic Institute & State University.

Lenth, R. V. (1989). Quick and easy analysis of unreplicated factorials. *Technometrics* **31**, 469-473.

Leon, R. V., Shoemaker, A. C. and Kackar, R. N. (1987). Performance measure independent of adjustment: An explanation and extension of Taguchi's signal to noise ratios. *Technometrics* **29**, 253-265.

Leon R. V. and Wu, C. F. J. (1992). A theory of performance measures on parameter design. *Statist. Sinica* **2**, 335-358.

Lucas, J. M. (1990). Achieving a robust process using response surface methodology. Du Pont Technical Report.

Nair, V. N. (1992). Taguchi's parameter design: A panel discussion. *Technometrics* **34**, 127-161.

Nair, V. N. and Pregibon, D. (1988). Comments on signal-to-noise ratios, performance criteria and transformation. *Technometrics* **30**, 24-29.

Pfaff, G. (1987). Pot core transformer processing experiment. *Proceeding of the Fifth Symposium on Taguchi Methods,* American Supplier Institute, Dearborn, MI, 427-438.

Shoemaker, A. C. and Tsui, K.-L. (1993). Response model analysis for robust design experiments. *Communications in Statistics: Simulation and Computation* **22**, 1037-1064.

Shoemaker, A. C. and Tsui, K.-L. and Wu, C. F. J. (1991). Economical experimentation methods for robust parameter design. *Technometrics* **33**, 415-427.

Taguchi, G. (1986). *Introduction to Quality Engineering: Designing Quality into Products and Processes.* Asian Productivity Organization, Tokyo, Japan.

Tsui, K.-L. (1993). A multi-step analysis procedure for robust design. Research Report No. J93-03, School of Industrial & Systems Engineering, Georgia Tech, Altanta.

Tsui, K.-L. (1996). A critical look at Taguchi's modeling approach for robust design. *J. Appl. Statist.*, in print.

Tsui, K.-L. and Li, A. (1994). Analysis of smaller- and larger-the-better robust design experiments. *International Journal of Industrial Engineering* 193-202.

Welch, W. J., Yu, T. K., Kang, S. M. and Sacks, J. (1990). Computer experiments for quality control by parameter design. *Journal of Quality Technology* **22**, 15-22.

Wu, C. F. J. (1987). Comments on performance measures independent of adjustment: An explanation and extension of Taguchi's signal-to-noise ratios. *Technometrics* **29**, 279-281.

Department of Industrial Engineering and Engineering Management, Hong Kong University of Science and Technology, Clear Water Bay, Hong Kong.

(Received July 1994; accepted July 1995)