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TESTS FOR BREAKS IN THE CONDITIONAL CO-MOVEMENTS OF ASSET RETURNS

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Abstract: We propose procedures designed to uncover structural breaks in the co-movements of financial markets. A reduced form approach is introduced that can be considered as a two-stage method for reducing the dimensionality of multivariate heteroskedastic conditional volatility models through marginalization. The main advantage is that one can use returns normalized by volatility filters that are purely data-driven and construct general conditional covariance dynamic specifications. The main thrust of our procedure is to examine change-points in the co-movements of normalized returns. The tests allow for strong and weak dependent as well as leptokurtic processes. We document, using a ten year period of two representative high frequency FX series, that regression models with non-Gaussian errors adequately describe their co-movements. Change-points are detected in the conditional covariance of the DM/US$ and YN/US$ normalized returns over the decade 1986-1996.

Key words and phrases: Change-point tests, conditional covariance, high-frequency financial data, multivariate GARCH models.

1. Introduction

There are many circumstances where one may expect that the co-movements between financial assets undergo fundamental changes. For example, portfolio holders may worry about the impact of the deregulation of an industry on their optimal allocation of assets which depends on conditional covariances (in a mean-variance setting). The deregulation may cause fundamental shifts in the (conditional) correlations across the asset holdings. Likewise, hedging strategies involving foreign exchange may be adversely affected by central bank policy shifts. Emerging markets is another example where the potential of breaks in co-movements may occur. The world equity markets liberalization and integration may represent an example of structural changes in the relationship of these markets. Similarly, the recent evidence of the Asian and Russian financial crises, transmitted across markets, have serious effects for investors, corporations and countries. The global character of financial markets presents an additional reason for examining the transmission of breaks and their effects in the co-movements between financial as well as real assets. Most financial asset pricing theories and models assume that covariances between assets are stable (possibly time varying) whereas more recent empirical approaches recognize the presence of time heterogeneity such as regime changes (e.g., Bollen, Gray and Whaley (2000)), institutional changes (e.g., Garcia and Ghysels (1998), Bekaert, Harvey and Lumsdaine (2002)) and extreme events (e.g., Hartmann, Straetmans and de Vries (2000)). Pastor and Stambaugh (2001) have also recently shown that structural breaks could contribute to the equity premium puzzle.

We propose procedures designed to uncover structural changes in multivariate conditional covariance dynamics of asset returns. The procedures are based on testing for breaks in the conditional correlations involving normalized returns which are defined as the returns standardized by the conditional variance process. Hence the conditional correlation is equivalent to the conditional covariance process of normalized returns that may exhibit a general form of dependence (e.g., $\phi-$ or $\alpha-$mixing) as well as heavy tails. We start from a multivariate dynamic heteroskedastic asset return process. Instead of trying to explore the co-movements via a parametric specification and test for structural change in the parameters, we adopt a reduced form approach which consists of testing for structural change in static or dynamic relationships involving marginalizations of the multivariate process. Our approach relates to a large class of multivariate ARCH-type models with constant or dynamic conditional correlation (see, for instance Bollerslev, Engle and Nelson (1994)). Although there is some loss of information when we look at the individual normalized returns and their relationships, these losses are offset by gains in reducing the overparameterized multivariate GARCH type models and by focusing on the conditional covariance specification. The latter is our focus in this paper. In addition this approach provides a simple and computationally efficient framework for testing and estimating the unknown (multiple) breaks in the co-movements of volatility and allows general forms of dependence as well as heavy tails without having to explicitly estimate their form.

The choice of standardized returns as an object of interest is motivated by both finance and statistics arguments. From the finance point of view, standardized returns relate to the fundamental measure of reward-to-risk consistent with conventional mean-variance analysis. The statistical arguments are a bit more involved. Our approach can be viewed as a two-stage method for reducing the dimensionality of multivariate heteroskedastic conditional volatility models to a framework involving returns normalized by purely data-driven volatility filters in the first stage, and cross products of normalized returns in the second stage. Recently, Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002) rely on a similar two-stage procedure to handle multivariate GARCH models. Their stages are both parametric whereas ours involve a first stage that is purely nonparametric. Our reduction approach does not aim to present alternative specification or estimation methods for multivariate GARCH models. Instead, we adopt this two stage approach as a method to perform change-point tests in multivariate heteroskedastic models. The approach here is semiparametric since the second stage can allow for general types of dependence, data-driven spot and quadratic volatility measures as well as leptokurtic or asymmetric distributions. More specifically, let $r_{(m),t} := \log p_t - \log p_{t-m}$ be the discretely observed time series of continuously compounded returns with m measuring the time span between discrete observations. We compute $X_{(m),t} := r_{(m),t}/\hat{\sigma}_{(m),t}$ involving purely data-driven estimators $\hat{\sigma}_{(m),t}$. Foster and Nelson (1996) proposed several rolling sample type estimators. Their setup applies to ARCH as well as discrete and continuous time SV models (which are in our application marginalizations of multivariate processes). In addition to the Foster and Nelson rolling volatility filters we also consider high-frequency volatility filters, following the recent work of Andersen, Bollerslev, Diebold and Labys (2001), Andreou and Ghysels (2002a), Barndorff-Nielsen and Shephard (2002a,b, 2003), among others. The data-driven measures of normalized returns provide the estimation of the first stage in multivariate heteroskedastic returns models. Moreover, keeping the first stage data-driven has the advantage that we do not specify, and therefore also not potentially misspecify, a parametric model for volatility. This may eliminate potential sources of misspecification and avoid erroneous inference on the presence of structural breaks. The second stage deals with the conditional covariance defined as the cross-product of normalized returns, say $Y_{12,(m),t} := X_{1,(m),t}X_{2,(m),t}$, for a pair of assets given by the vector $(1,2)'$. This process may exhibit constant, weak or strong dependence (as in multivariate constant or dynamic correlation GARCH and Factor models, respectively) as well as a general functional form driven by a heavy tailed distribution. In addition, auxiliary regression models for normalized returns are employed to study the homogeneity of their co-movements. The simulation and empirical results in the paper show that standardized returns, using various volatility filters, are in most cases non-Gaussian with different types of temporal dependence structure. The paper extends the application of recent change-point tests in Kokoszka and Leipus (1998, 2000) and Lavielle and Moulines (2000) to the conditional covariance of Multivariate GARCH (M-GARCH) models, using the above two-stage procedure for detecting breaks in the co-movements of normalized returns.

The paper is organized as follows. In Section 2 we discuss the general multivariate conditional volatility models and the transformations of the data that form the basis of the testing procedure. Section 3 discusses recent change-point tests, developed in a univariate context, and a method to apply them to the conditional covariance processes of multivariate heteroskedastic models. The fourth section presents a brief Monte Carlo experiment that examines the statistical properties of normalized returns and provides a justification for the testing strategies adopted. The size and power of the aforementioned tests are also investigated. In the empirical section we document using a ten year period of two representative high frequency FX series, YN/US$ and DM/US$, that the conditional covariance specified by regression models of daily standardized returns with non-Gaussian errors adequately describe their co-movements. The main thrust of our procedure is then to examine breaks in the co-movements of normalized returns using CUSUM and least-squares methods for detecting and dating the change-points. A final section concludes the paper.

2. Models and Filters

It has long been recognized that there are gains from modeling volatility co-movements. In practice one stumbles on the obvious constraint that any multivariate model is hopelessly overparameterized if one does not impose any type of restriction (see for instance, Engle (2001) for some of the open questions in multivariate volatility models). Bollerslev, Engle and Nelson (1994) provide an elaborate discussion of various multivariate ARCH type models and review the different restrictions which have been adopted to make multivariate volatility models empirically feasible. Ghysels, Harvey and Renault (1996) discuss various multivariate SV models, both in discrete and continuous time. In this section we describe the classes of multivariate heteroskedastic models that fall within the context of our statistical procedures for change-point tests in the dynamic co-movements of asset returns. Broadly speaking there are two classes of multivariate volatility models, both being among the most widely applied parametric specifications. These are (1) multivariate factor models, see for instance Diebold and Nerlove (1989), Engle, Ng and Rotschild (1990), Harvey, Ruiz and Shephard (1994), Ng, Engle and Rotschild (1992) and many others, and (2) the conditional correlation models, see for instance Bollerslev, Engle and Wooldridge (1988), Bollerslev (1990), Bolleslev, Engle and Nelson (1994) and more recently Engle (2002), Engle and Sheppard (2001) and Tse and Tsui (2002). Since the statistical procedures adopted here share many features with the latter, we devote the first subsection to the conditional correlation volatility specification. The second subsection describes various volatility filters which are adopted for dynamic heteroskedastic series.

2.1. Multivariate conditional correlation volatility models

The statistics developed in this paper apply to a two-step procedure that shares several features with the recent work on Dynamic Conditional Correlation (henceforth DCC) of Engle (2002), Engle and Sheppard (2001) and Tse and Tsui

(2002). The appeal of these is that they feature the flexibility and simplicity of univariate ARCH models but not have the complexity of typical multivariate specifications. This decomposition also presents an advantage for change-point detection in multivariate heteroskedastic settings, discussed further in Section 3. The statistical inference procedures proposed apply to several multivariate specifications given that the conditional covariance process satisfies some general regularity conditions. It will be convenient to start with a discrete time framework, and to set notation we assume that an n-vector of returns R_t is observed. In the empirical applications n will be equal to 2, but our techniques extend to $n > 2$. Consider the ratio $X_{i,t} := r_{i,t}/\sigma_{i,t}$, where $r_{i,t}$ and $\sigma_{i,t}$ are the return and conditional volatility (standard deviation) of the i^{th} return process, respectively, using the *univariate* filtration of each series separately. Then the conditional correlation between pairs of assets, e.g., $(1,2)'$ is: $\rho_{12,t} = E_{t-1}(X_{1,t}X_{2,t}) := E_{t-1}(Y_{12,t})$, where we denote $Y_{12,t} := X_{1,t}X_{2,t}$. The original specification of Bollerslev (1990) assumed that $\rho_{12,t} := \rho_{12}$, yielding a CCC model, i.e., a Constant Conditional Correlation multivariate specification. It was noted that the CCC specification offered many computational advantages, but the assumption of constant ρ_{12} does not enjoy much empirical support.

The procedures proposed in this paper also involve the $X_{1,t}$, $X_{2,t}$ and $Y_{12,t}$ processes. However, these processes are obtained in a much more general context not involving a parametric specification for the conditional standard deviation $\sigma_{i,t}$ for $i = 1, 2$. The DCC specification assumes that $\sigma_{i,t}$ follows a GARCH(1,1) model. We adopt a purely data-driven specification for $\sigma_{i,t}$, and this has several advantages. First this approach covers processes more general than the GARCH specification, some of which can account for asymmetries as well as jumps (given the results in Foster and Nelson (1996), Andersen, Bollerslev, Diebold and Labys (2001) and Andreou and Ghysels (2002a)). The purely data-driven first stage also has the advantage that we do not potentially misspecify the parametric model for volatility. Moreover, this approach may avoid some potential sources of misspecification and erroneous inference on the presence of structural breaks. This is related to the second advantage of the method proposed in that it yields a semi-parametric setup for the second stage of the test procedure that also allows for general innovation distributions.

In the remainder of this subsection we discuss only the basic underpinnings of filtering $\sigma_{i,t}$. The notation is simplified here by dropping the subscript i pertaining to a particular return series, i.e., instead of $r_{i,t}$ we simply write r_t because we adopt mainly a univariate framework. The computation of r_t/σ_t with data-driven σ_t is valid in a diffusion context as well as discrete time processes, such as various ARCH type models including GARCH, EGARCH, SV and other specifications. The setup is deliberately closely related to the work of Foster and

Nelson (1996) on rolling sample volatility estimators. Consider the following discrete time dynamics:

$$r_{(m),t} = \mu_{(m),t} m^{-1} + M_{(m),t} - M_{(m),t-m} \equiv \mu_{(m),t} m^{-1} + \Delta_{(m)} M_{(m),t} \tag{2.1}$$

which correspond to the so called Doob-Meyer decomposition of the m horizon returns into a predictable component $\mu_{(m),t}$ and a local martingale difference sequence. The decomposition is a natural starting point when returns are generated by a standard diffusion process with stochastic volatility. The decomposition in (2.1) is also the starting point for discrete time ARCH type processes. Conditional expectations and variances with respect to the (univariate) filtration $\{\mathcal{F}_{(m),t}\}$ will be denoted as $E_{(m),t}(\cdot)$ and $\mathrm{Var}_{(m),t}(\cdot)$ respectively, whereas unconditional moments follow a similar notation, $E_{(m)}(\cdot)$ and $\mathrm{Var}_{(m)}(\cdot)$. Consequently

$$\mathrm{Var}_{(m),t}(r_{(m),t}) \equiv E[(\Delta_{(m)} M_{(m),t})^2 | \mathcal{F}_{(m),t}] = \sigma^2_{(m),t} m^{-1}, \tag{2.2}$$

where $\sigma^2_{(m),t}$ measures the conditional variance per unit of time. We consider various data-driven estimators for $\sigma^2_{(m),t}$ which can generically be written as:

$$\hat{\sigma}^2_{(m),t} = \sum\nolimits_{\tau=1}^{n_L} w_{(\tau-t)} (r_{(m),t+1-\tau} - \hat{\mu}_{(m),t})^2, \tag{2.3}$$

where $w_{(\tau-t)}$ is a weighting scheme, n_L is the lag length of the rolling window and $\hat{\mu}_{(m),t}$ is a (rolling sample) estimate of the drift. The optimal window length and weights are discussed in Foster and Nelson (1996) and Andreou and Ghysels (2002a), and applied in the empirical section.

2.2. Transformations of returns using data-driven volatilities

The test statistics discussed in the next section are based on functions of normalized returns computed as $(r_{(m),t} - \hat{\mu}_{(m),t})/\hat{\sigma}_{(m),t}$, for some estimator of $\hat{\mu}_{(m),t}$ and $\hat{\sigma}_{(m),t}$, i.e., some sampling frequency m and weighting scheme $w_{(\tau-t)}$ in (2.3). The empirical setting that will be used involves very short spans of data with high frequency sampling. We can deal with the local drift either by estimating it as a local average sum of returns or, following the arguments in Merton (1980) among others, ignore any possible drift and set it to zero, i.e., $\hat{\mu}_{(m),t} \equiv 0$. For simplicity we adopt the latter and set the drift to zero.

The setup in (2.1) and (2.2) is the same as in Foster and Nelson (1996), who derive a continuous record asymptotic theory which assumes that a fixed span of data is sampled at ever finer intervals. The basic intuition driving the results is that normalized returns, $r_{(m),t}/\sigma_{(m),t}$, over short intervals are *approximately* i.i.d. with zero conditional mean and finite conditional variance and have regular tail behavior, which make the application of central limit theorems possible. Foster and Nelson impose several fairly mild regularity conditions such that the local behavior of the ratio $r_{(m),t}/\sigma_{(m),t}$ becomes approximately i.i.d. with fat tails (and eventually Gaussian for large m). In their setup local cuts of the data exhibit a relatively stable variance, which is why $\hat{\sigma}_{(m),t}$ catches up with the latent true $\sigma_{(m),t}$ with judicious choices of the weighting scheme and, in particular, the data window chosen to estimate the local volatility. The tests allow for some local dependence in the data and do *not* rely on normality of the ratio $r_{(m),t}/\hat{\sigma}_{(m),t}$. The empirical evidence of the normality of $r_{(m),t}/\hat{\sigma}_{(m),t}$ is mixed at the daily level at least. Zhou (1996) and Andersen, Bollerslev, Diebold and Labys (2000) report near-normality for daily sampling frequencies. We find that different classes of volatility filters yield different distributional properties for the normalized returns process, $X_{(m),t}$.

A number of alternative volatility filters, $\hat{\sigma}_{i,(m),t}$, are considered below which differ in terms of estimation method, sampling frequency and information set (further evaluated in Foster and Nelson (1996), Andersen and Bollerslev (1998), Andersen, Bollerslev, Diebold and Labys (2001) and Andreou and Ghysels (2002a)). These data-driven variance filters belong to two classes. The first include: (i) Exponentially Weighted Moving Average Volatility, defined following the industry standard introduced by J. P. Morgan (see Riskmetrics Manual (1995)) as $\hat{\sigma}_{RM,t} = \lambda\hat{\sigma}_{RM,t-1} + (1-\lambda)\, r_t^2,\ t = 1, \ldots, T_{days}$, where $\lambda = 0.94$ for daily data, r_t is the daily return, and T_{days} is the number of trading days; (ii) One-sided Rolling daily window Volatility defined as $\hat{\sigma}_{RV,t} = \sum_{j=1}^{n_L} w_j r_{t+1-j}^2,\ t = 1, \ldots, T_{days}$, where n_L is the lag length of the rolling window in days. In our simulations we consider $n_L = 26$ and 52 days to conform with the optimality in Foster and Nelson, and the common practice of taking (roughly) one month worth of data (see e.g., Schwert (1989) among others). These interday volatilities are denoted as $\hat{\sigma}_{i,t}$ where $i = RM,\ RV26,\ RV52$. The second class of intraday volatility filters is based on the quadratic variation of returns (see Andreou and Ghysels (2002a) for more details). These include: (i) One-day Quadratic Variation of the process (also called Integrated Volatility, e.g., Andersen and Bollerslev (1998)) is the sum of squared log returns $r_{(m),t}$ for different values of m, with $\hat{\sigma}_{QV1,t} = \sum_{j=1}^{m} r_{(m),t+1-j/m}^2,\ t = 1, \ldots, n_{days}$ (for the 5-minute sampling frequency, the lag length is $m = 288$ for financial markets open 24 hours per day, e.g., FX markets); (ii) One-day Historical Quadratic Variation (introduced in Andreou and Ghysels (2002a)), defined as the sum of m rolling $QV1$ estimates $\hat{\sigma}_{HQV1,t} = 1/m \sum_{j=1}^{m} \hat{\sigma}_{QV1,(m),t+1-j/m},\ t = 1, \ldots, T_{days}$. These intraday volatilities are denoted as $\hat{\sigma}_{i,t}$ where $i = QVk, HQVk$, for window lengths $k = 1, 2, 3,$

in the 5-minute sampling frequency case. For window lengths $k > 1$ the intraday volatility filters $(H)QVk$ are simple averages of $(H)QV1$ for k days.

3. Tests for Structural Breaks in Co-Movements

There is a substantial literature on testing for the presence of breaks in i.i.d. processes and more recent work in the context of linearly dependent stochastic processes (see for instance, Liu, Wu and Zidek (1997) and Bai and Perron (1998)). Nevertheless, high frequency financial asset returns series are strongly dependent processes satisfying $\beta-$mixing. Chen and Carrasco (2001) provide a comprehensive analysis of such univariate processes and Bussama (2001) and Chen and Hansen (2002) have shown that multivariate ARCH and diffusion processes are also β-mixing. This result precludes the application of many tests for structural breaks that require stronger mixing conditions. Following Kokoszka and Leipus (1998, 2000) and Lavielle and Moulines (2000), we explore recent advances in the theory of change-point estimation for strongly dependent processes. These papers have shown the consistency of CUSUM and least squares type change-point estimators, respectively, for detecting and dating change-points. The tests are not model-specific and apply to a large class of weakly and strongly dependent (e.g., ARCH and SV type) specifications. So far only limited simulation and empirical evidence has been reported about these tests. Andreou and Ghysels (2002b) enlarged the scope of applicability by suggesting several improvements that enhance the practical implementation of the proposed tests. They also find, via simulations, that the VARHAC estimator proposed by den Haan and Levin (1997) yields good properties for the CUSUM-type estimator of Kokoszka and Leipus (2000).

The Lavielle and Moulines (2000) and Kokoszka and Leipus (2000) studies can handle univariate processes while we investigate multivariate processes via the two-step setup. It is demonstrated that the two-stage approach adopted here for multivariate models can be considered as a simple reduced form, and computationally efficient, method for the detection of structural breaks tests in multivariate heteroskedastic settings. The procedures proposed apply to the empirical process $Y_{12,t} := X_{1,t}X_{2,t}$ for pairs of assets normalized returns of M-GARCH type models, where $X_{i,t} := r_{i,t}/\sigma_{i,t}$, $i = 1, 2$, is obtained via the application of a data-driven filter described in the previous section. The β-mixing property of multivariate GARCH and diffusion processes (Bussamma (2001), Chen and Hansen (2002)) implies that $Y_{12,t}$ is β-mixing as well. This is valid for the M-GARCH with dynamic conditional correlation specifications. For instance, according to the M-GARCH-DCC (Engle (2002)), $Y_{12,t}$ has a GARCH specification which implies β-mixing, the exception being the M-GARCH-CCC according to which $Y_{12,t}$ is assumed to be constant. Last but not least, we note that in dynamic correlation M-GARCH models quadratic transformations, such as $|Y_{12,t}|^d$ $d = 1, 2$, are also β-mixing since they are measurable functions of mixing processes which are β-mixing and of the same size (see White (1984, Theorem 3.49 and Proposition 3.23)).

The analysis focuses on the bivariate case for ease of exposition. This two-step approach can be extended to the n asset M-GARCH framework for which $n(n-1)/2$ cross-covariances, $Y_{ij,t}$, would be the processes for testing the change-point hypothesis in pairs of assets. It is worth noting that when n gets large, this framework becomes useful if we impose some additional restrictions. For instance in the M-GARCH-CCC model, when n gets large, we can test the null hypothesis of joint homogeneity in the correlation coefficients in the pairs of normalized returns, ρ_{ij}, versus the alternative that there is an unknown change-point in the any of these cross-correlations. A similar approach for $n-$dependent processes can be found in Horváth, Kokoszka and Steinebach (1999) and can be adapted to the conditional covariances of an M-GARCH-CCC model. In the remainder of this section we discuss the specifics of the testing procedures.

3.1. CUSUM type tests

Without an explicit specification of a multivariate ARCH, the tests discussed in this section will examine whether there is evidence of structural breaks in the data generating process of $Y_{12,t}$. To test for breaks, Kokoszka and Leipus (1998, 2000) consider

$$U_N(k) = \left((1/\sqrt{N}) \sum_{j=1}^{k} Z_j - (k/(N\sqrt{N})) \sum_{j=1}^{N} Z_j\right) \tag{3.1}$$

for $0 < k < N$, where $Z_t = |Y_{12,t}|^d$, $d = 1, 2$, in (3.1) represent the absolute and squared normalized returns in an ARCH(∞) process. When the conditional covariance process exhibits an ARCH-type specification, like in most dynamic conditional correlation M-GARCH models, we need not specify the explicit functional form of $Y_{12,t}$. Kokoszka and Leipus (1998, 2000) assume that ARCH(∞) processes are (i) stationary with short memory, i.e., the coefficients decay exponentially fast, and (ii) the errors are not assumed Gaussian but do have a finite fourth moment. Horváth (1997) and Kokoszka and Leipus (1998) show that (3.1) holds if the process $Z_t := Y_{12,t}$ is linearly dependent. The above moment conditions need also apply to M-GARCH processes. The CUSUM type estimators are

$$\hat{k} = \min\{k : |U_N(k)| = \max_{1 \le j \le N} |U_N(j)|\}. \tag{3.2}$$

The estimate $\hat{k}$ is the point at which there is maximal sample evidence for a break in the Z_t process. To decide whether there is actually a break, one has also to derive the asymptotic distribution of $\sup_{0\leq k\leq N} U_N(k)$, or related processes such as $\int_0^1 U_N^2(t)dt$. Moreover, in the presence of a single break, $\hat{k}$ is a consistent estimator of k^*. Under the null hypothesis of no break,

$$U_N(k) \rightarrow_{D[0,1]} \sigma_Z B(k), \tag{3.3}$$

where $B(k)$ is a Brownian bridge and $\sigma_Z^2 = \sum_{j=-\infty}^{\infty} Cov(Z_j, Z_0)$. Consequently, using an estimator $\hat{\sigma}_Z$, one can establish that under the null:

$$\sup\{|U_N(k)|\}/\hat{\sigma}_Z \rightarrow_{D[0,1]} \sup\{B(k) : k\epsilon[0,1]\}, \tag{3.4}$$

which yields a Kolmogorov-Smirnov type asymptotic distribution. Further details about the computation of the statistics and its application to multiple breaks in a univariate GARCH context can be found in Andreou and Ghysels (2002b).

3.2. Least squares type tests

Liu, Wu and Zidek (1997) and Bai and Perron (1998) have proposed a least squares estimation procedure to determine the number and location of breaks in the mean of linear processes with weakly dependent errors. Their key result uses a Hájek-Rényi inequality to establish the asymptotic distribution of the test procedure. Recent work by Lavielle and Moulines (2000) has greatly increased the scope of testing for multiple breaks. They obtain similar inequality results for weakly, as well as strongly, dependent processes. The number of breaks is estimated via a penalized least-squares approach similar to Yao (1988). In particular, Lavielle and Moulines (2000) show that an appropriately modified version of the Schwarz criterion yields a consistent estimator of the number of change-points. In the present analysis we apply this test to the model

$$Y_{12,t} = \mu_k^* + \varepsilon_t, \quad t_{k-1}^* \leq t \leq t_k^*, \quad 1 \leq k \leq r, \tag{3.5}$$

where $t_0^* = 0$ and $t_{r+1}^* = T$, the sample size. The indices of the breakpoint and mean values μ_k^*, $k = 1, \ldots, r$, are unknown. It is worth recalling that $Y_{12,t}$ is a generic stand-in process. In our application, (3.5) applies to the cross-products of normalized returns for examining the change-point hypothesis in the conditional covariance of M-GARCH-CCC and -DCC type models.

For dynamic conditional correlation models, (3.5) can be augmented to

$$Y_{12,t} = \theta_{12} + \eta_{12} Y_{12,t-1} + v_{12,t}. \tag{3.6}$$

When the M-GARCH conditional correlation is assumed constant, or when dealing with a single observed factor model (e.g., the market CAPM) with constant correlation, another auxiliary equation that may yield power for testing the structural breaks hypothesis is the regression between normalized returns, e.g., $X_{1,t} = \theta'_{12} + \eta'_{12}X_{2,t} + v_{12,t}$. Note that this regression is not strictly equivalent to (3.5) for the conditional covariance that is derived from the M-GARCH-CCC reduction approach. Nevertheless, it can be considered as another auxiliary regression that relates to the conditional co-movements between assets in factor models, as well as most conditional mean asset pricing theories.

A useful example of this approach can be considered in the context of the one factor model that is used to model the market CAPM model. Let $r_{M,t}$ and $r_{i,t}$ be the demeaned returns on the market (indexed by M) and on the individual firm stock i at time t :

$$r_{M,t} = \sigma_{M,t}u_{M,t}, \tag{3.7}$$

$$r_{i,t} = \beta_{i,t}r_{M,t} + \sigma_{i,t}u_{i,t}, \tag{3.8}$$

where $u_{M,t}$ and $u_{i,t}$ are uncorrelated i.i.d.$(0,1)$ processes, $\sigma_{M,t}, \sigma_{i,t}$ and $\beta_{i,t}$ are, respectively, the conditional variance of $r_{M,t}$, the firm specific variance of $r_{i,t}$, and the conditional beta of $r_{i,t}$ with respect to $r_{M,t}$. Beta is expressed as

$$\beta_{i,t} = E_{t-1}(r_{i,t}r_{M,t})/E_{t-1}(r^2_{M,t}) := \sigma_{iM,t}/\sigma^2_{M,t}. \tag{3.9}$$

In the market CAPM equation (3.8), we divide by $\sigma_{i,t}$ and write beta explicitly to obtain $r_{i,t}/\sigma_{i,t} = (\sigma_{iM,t}/(\sigma_{M,t}\sigma_{i,t})(r_{M,t}/\sigma_{M,t})) + (\sigma_{i,t}z_{i,t})/\sigma_{i,t}$. If we define the normalized returns by $X_{i,t}$ and $X_{M,t}$, then the following regression type model arises: $X_{i,t} = (\sigma_{iM,t}/(\sigma_{M,t}\sigma_{i,t}))X_{M,t} + z_{i,t}$, or

$$X_{i,t} = \rho_{iM,t}X_{M,t} + z_{i,t}, \tag{3.10}$$

where $\rho_{iM,t}$ represents the conditional correlation between the returns of the two assets. Two interesting cases arise in the context of (3.10). If $\rho_{iM,t} = \rho_{iM}$, then constant conditional correlation implies the process (3.10) is $\phi-$mixing. If $\rho_{iM,t}$ is a dynamic conditional correlation, then (3.10) is $\beta-$mixing. In both cases the Lavielle and Moulines test can be applied. Note that the above example is restricted to observable factors and can be extended to n risky assets to obtain n regressions of normalized returns with the risk adjusted market portfolio. The change-point could be performed on each equation (3.10) to assess the stability of the co-movements of risky stocks with the market portfolio.

The Lavielle and Moulines tests are based on the least-squares computation

$$Q_T(t) = \min_{\mu^*_k, k=1,\ldots,r} \sum\nolimits_{k=1}^{r+1} \sum\nolimits_{t=t_{k-1}+1}^{t_k} (Y_{12,k} - \mu_k)^2. \tag{3.11}$$

Estimation of the number of break points involves the use of the Schwarz or Bayesian information criterion (BIC), and hence a penalized criterion $Q_T(t)+\beta_T r$ where $\beta_T r$ is a penalty function to avoid over-segmentation with r the number of changes and $\{\beta_T\}$ a decreasing sequence of positive real numbers. We examine the properties of this test using both BIC and the information criterion proposed in Liu, Wu and Zidek (1997) (denoted as LWZ). It is shown under mild conditions that the change-point estimator is strongly consistent with rate of convergence T.

4. Monte Carlo Design and Results

In this section we discuss the Monte Carlo study which examines the properties of normalized returns in univariate and multivariate heteroskedastic parameterizations, as well as the properties of the Kokoszka and Leipus (1998, 2000) and Lavielle and Moulines (2000) change-point tests applied in a multivariate heteroskedastic setting. The design and results complement the findings of Andreou and Ghysels (2002a,b), who propose extensions of the continuous record asymptotic analysis for rolling sample variance estimators and examine the aforementioned tests for testing breaks in the dynamics of univariate volatility models.

4.1. Simulation design

The simulated returns processes are generated from the following two types of DGPs: (i) a univariate GARCH process with Normal and Student's t errors, and (ii) a multivariate GARCH process with constant correlation (M-GARCH-CCC) (Bollerslev (1990)), as well as dynamic correlation such as the *vech* diagonal specification proposed in Bollerslev, Engle and Wooldridge (1988) (M-GARCH-VDC). The choice of the M-GARCH-CCC and M-GARCH-VDC models is mainly due to their simplicity and parsimony for simulation and parameterization purposes. Moreover, the former multivariate design is most closely related to the univariate GARCH for which the Kokoszka and Leipus (2000) test has been derived. More specifically, the DGPs examined are the following.
(i) The univariate GARCH process is:

$$r_{q,t} = u_{q,t}(\sigma_{q,t})^{1/2}, \;\; \sigma_{q,t} = \omega_q + a_q r^2_{q,t-1} + \beta_q \sigma_{q,t-1}, \tag{4.1}$$

where $r_{q,t}$ is the returns process generated by the product of the error $u_{q,t}$, which is i.i.d.$(0,1)$ with Normal or Student's t distribution, and the volatility process $\sigma_{q,t}$ that has a GARCH(1,1) specification. The process without change points is denoted by $q = 0$, whereas a break in any of the parameters of the process is symbolized by $q = 1$ to denote the null and the alternative hypotheses, respectively, outlined below.
(ii) The multivariate GARCH process for a pair of assets denoted by $(1,2)$ is

$$\begin{aligned} r_{1,q,t} &= r_{1,q,t}(h_{11,q,t})^{1/2} + u_{2,q,t}h_{12,q,t}, \\ r_{2,q,t} &= r_{2,q,t}(h_{22,q,t})^{1/2} + u_{1,q,t}h_{12,q,t}, \quad t = 1,\ldots,T \quad \text{and} \quad q = 0,1, \end{aligned} \tag{4.2}$$

where $r_{1,q,t}$ and $r_{2,q,t}$ are the returns processes that are generated by $u_{1,q,t}$ and $u_{2,q,t}$, i.i.d.$(0,1)$ processes, and M-GARCH conditional variances

$$\begin{aligned} h_{11,q,t} &= \omega_{11,q} + a_{11,q} r_{1,q,t-1}^2 + \beta_{11,q} h_{11,q,t-1}, \\ h_{22,q,t} &= \omega_{22,q} + a_{22,q} r_{2,q,t-1}^2 + \beta_{22,q} h_{22,q,t-1}. \end{aligned} \tag{4.3}$$

The conditional covariance in the M-GARCH-CCC (Bollerslev (1990)) is given by

$$h_{12,q,t} = \rho_{12,q} (h_{11,q,t} h_{22,q,t})^{1/2}. \tag{4.4}$$

Similarly the conditional covariance in the M-GARCH-VDC (Bollerslev, Engle and Wooldridge (1988)) is given by

$$h_{12,q,t} = \omega_{12,q} + a_{12,q} r_{1,q,t-1} r_{2,q,t-1} + \beta_{12,q} h_{12,q,t-1}. \tag{4.5}$$

The models used in the simulation study are representative of financial markets data with a set of parameters that capture a range of degrees of volatility persistence measured by $\delta = a + \beta$. The vector parameter (ω, a, β) in (4.1) describes the following data generating processes: DGP1 has $(0.4, 0.1, 0.5)$ and DGP2 has $(0.1, 0.1, 0.7)$, characterized by low and high volatility persistence, respectively. In order to control the multivariate simulation experiment, the volatility processes of the M-GARCH equations in (4.3) are assumed to have the same parameterization. The sample sizes $N = 500$ and 1000 are chosen so as to examine not only the asymptotic behavior but also the small sample properties of the tests for realistic samples in financial time series. For simplicity and conciseness the simulation design is restricted to the bivariate case, it can be extended to $n > 2$ assets where the tests are applied to pairs just as in the bivariate model.

The models in (i) and (ii) without breaks ($q = 0$) denote the processes under the null hypothesis for which the simulation provides evidence for the size of the Kokoszka and Leipus and Lavielle and Moulines tests. Under the alternative hypothesis the returns process is assumed to exhibit breaks. Four cases are considered to evaluate the power of the tests. The simulation study focuses on the single change-point hypothesis but can be extended to the multiple breaks framework (see for instance, Andreou and Ghysels (2002b)). In the context of (4.1) we study breaks in the conditional variance $h_{q,t}$, which can also be thought of as permanent regime shifts in volatility at change points πN ($\pi = 0.3, 0.5, 0.7$). Such breaks may have the following sources: H_1^A : a change in the volatility dynamics, β_q; H_1^B : a change in the intercept, ω_q; H_1^C : a change in the conditional correlation, given by $\rho_{12,q}$ in (4.4) or H_1^D : $\omega_{12,q}$ or $\beta_{12,q}$ in (4.5).

The simulation investigation is organized as follows. First we examine some of the probabilistic properties of the normalized returns series generated from univariate and multivariate GARCH models. Second we investigate the performance of the Kokoszka and Leipus and Lavielle and Moulines tests using the multivariate normalized returns framework. We test for breaks in the cross-product of normalized returns or the regression of normalized returns. The simulation, as well as the empirical analysis, is performed using the GAUSS programming language.

4.2. The standardized returns processes

The statistical properties of daily returns standardized by volatility filters are discussed in the context of the univariate and bivariate dynamic heteroskedastic structures described above. For intraday volatility filters and for the purpose of simulation and parameter selection we take the univariate representation of each GARCH process for alternative sampling frequencies following Drost and Werker (1996, Corollary 3.2), who derive the mappings between GARCH parameters corresponding to processes with $r_{(m),t}$ sampled with different values of m. Obviously the Drost and Werker formulae do not apply in multivariate settings but they are used here for the marginal process, producing potentially an approximation error as the marginal processes are not exactly weak GARCH(1,1). Using the estimated GARCH parameters for daily data with $m = 1$, one can compute the corresponding parameters $\omega_{(m)}$, $\alpha_{(m)}$ and $\beta_{(m)}$, for any other frequency m. The models used for the simulation study are representative of the FX financial markets, popular candidates of which are taken to be returns on DM/US$, YN/US$ exchange rates. We take the daily results of Andersen and Bollerslev (1998) and compute the implied GARCH(1,1) parameters $\omega_{(m)}$, $\alpha_{(m)}$ and $\beta_{(m)}$ for 1-minute and 5-minutes frequency, $m = 1440$ and 288 respectively, using the software available from Drost and Nijman (1993).

The normalized returns transformation is the process of interest following the discussion in Section 2. According to the univariate GARCH process (4.1), the standardized returns process $X_{i,(m)} := r_{i,(m),t}/\sigma_{i,(m),t}$ is by definition i.i.d.$(0, 1)$. The 'true' standardized returns of the univariate GARCH is given for the 1-minute sampling frequency and the corresponding parameters found in Andreou and Ghysels (2002a). Quadratic variation intraday estimators are specified by aggregating the 'true' squared returns process at 5-, 30- and 60-minutes sampling frequencies. The remaining volatility filters are the spot volatilities which are specified here using daily frequencies. Evaluation of how well the returns standardized by the data-driven volatility filters approximate the true parametric structure is based on the contemporaneous Mean Square and Absolute Errors (MSE and MAE) using as benchmark the MSE (and MAE) of $X_{QV1,t}$. We also evaluate the statistical properties of these ratio transformations by testing their normality assumption using the Jarque and Bera (1980) test, as well as whether they adequately capture the nonlinear dynamics by testing for any remaining ARCH (Engle (1982)) in $X_{i,t}$. According to the bivariate GARCH process with constant or dynamic correlation, (4.4) or (4.5) respectively, the normalized returns is expected to be a dependent process.

Table 1. Monte Carlo Simulations of MSE and MAE ratios, normality and second-order dependence test results for daily FX $X(i)$ = Returns/Volatilities (i) of YN/US$ calculated at 5-minute frequency.

	Jarque Bera Normality Test		ARCH Test	
	N-GARCH JB	t -GARCH JB	N-GARCH ARCH(5)	t -GARCH ARCH(5)
X(RM)	113.8	3254	0.921	0.623
	(0.000)	(0.000)	(0.536)	(0.701)
X(RV26)	266.7	2735	0.921	0.623
	(0.000)	(0.000)	(0.536)	(0.701)
X(RV52)	1368	13587	0.705	0.599
	(0.000)	(0.000)	(0.730)	(0.730)
X(QV1)	2.132	4.169	0.986	1.030
	(0.447)	(0.215)	(0.496)	(0.491)
X(QV2)	6.514	19.15	1.262	1.886
	(0.190)	(0.010)	(0.392)	(0.210)
X(QV3)	24.48	100.9	1.354	2.063
	(0.006)	(0.000)	(0.366)	(0.166)
X(HQV1)	388.4	9056	1.354	1.324
	(0.018)	(0.000)	(0.358)	(0.470)
X(HQV2)	555.3	26687	1.550	1.367
	(0.000)	(0.000)	(0.369)	(0.474)
X(HQV3)	1253	38579	1.350	1.155
	(0.000)	(0.000)	(0.407)	(0.497)

Note: The simulation design is described in Section 3. We consider normal and Student's t (with 6 degrees of freedom) GARCH processes. The volatility filters are defined in the end of Section 2.2. The standardized returns are tested for normality using the Jarque-Bera (JB) test. We examine any remaining second-order temporal dependence in standardized returns using the ARCH test with the corresponding lag length in parentheses. Similar results were obtained for alternative lag lengths. p-values are reported below the test statistics in the parentheses. The total sample size is 2500 which is adjusted for the subsample of 2250 due to the standardized returns by rolling volatilities.

The simulation results in Table 1 summarize the statistical properties of the daily returns standardized by the alternative volatility filters (defined in Section 2.2) with respect to their distributional and temporal dependence dynamic properties. We focus on the univariate GARCH process since it is expected that the normalized returns from an M-GARCH process will exhibit second-order dependence due to unmodelled conditional covariance dynamics. The normality test results show that in the case of the normal GARCH process, there is general simulation evidence that does not support the normality hypothesis for most standardized returns series (at the 5% significance level), except for $X_{QV1,t}$ and $X_{QV2,t}$. Similarly, under the more realistic assumption of a t-GARCH, arising from heavy-tailed high-frequency data, we do not find supportive evidence of the normality hypothesis in all series except $X_{QV1,t}$. Table 1 also presents the simulation results from testing any remaining ARCH effects in normalized returns. We find evidence in favor of no remaining second-order dynamics in all risk-adjusted returns by interday and intraday volatility filters, under both normal and Student's t univariate GARCH processes. The results present evidence that univariate returns process normalized by optimal volatility filters yield an approximately independent series, with a distribution that has different tail behavior depending on the standardizing filter employed.

4.3. Simulation results of change-point tests

In Section 2 we discussed the reduced form approach adopted for M-GARCH models. The first stage involves the univariate specification and estimation of conditional variance dynamics which yield the normalized returns process for each asset, $X_{1,t}$ and $X_{2,t}$. The second stage involves the specification of the conditional covariance dynamics. For M-GARCH processes the conditional covariance is specified as the cross-product of pairs of normalized returns for assets 1 and 2 given by $Y_{12,t} = X_{1,t}X_{2,t}$. The equations for $Y_{12,t}$, which we use for change-point testing, are given by (3.5) and (3.6) which represent the constant and dynamic conditional correlation of M-GARCH-CCC and M-GARCH-VDC models, respectively. The specification in (3.6) for the conditional correlation as well its ARMA generalizations have been discussed in Engle (2002) and Tse and Tsui (2002). The simulation test results focus on $N = 1000$ and $\pi = 0.5$ for conciseness purposes.

The simulation results for the properties of the Kokoszka and Leipus (K&L) test are reported in Table 2. We consider the cross product of normalized returns $X_{1,t}X_{2,t}$ (using volatility estimators) as well as the 'true' simulated cross product of normalized returns given by $u_{1,t}u_{2,t}$ in (4.2). We focus on the $X_{RV26,t}$ and $X_{RM,t}$ series which are applicable in a broader sense given their daily sampling frequency, as well as the relationship of the RiskMetricks with IGARCH models. Note that the empirical analysis considers all volatility filters discussed in Section 2.2. The representative simulation results in Table 2 show that although the K&L test has good size properties for simulated cross product of normalized returns, $u_{1,t}u_{2,t}$, it is seriously undersized for the estimated normalized returns, $X_{1,t}X_{2,t}$,

Table 2. Size and power of the Kokoszka and Leipus (2000) test for a change-point in the co-movements of normalized returns.

Statistic: $U\max/\widehat{\sigma}_{VARHAC}$ Sample: $N = 1000$ Change-point timing: $\pi = 0.5$										
Processes	True errors			$X_1(RV26){*}X_2(RV26)$			$X_1(RM){*}X_2(RM)$			
Transformations	$u_{1,t}u_{2,t}$	$(u_{1,t}u_{2,t})^2$	$\|u_{1,t}u_{2,t}\|$	$X_{1,t}X_{2,t}$	$(X_{1,t}X_{2,t})^2$	$\|X_{1,t}X_{2,t}\|$	$X_{1,t}X_{2,t}$	$(X_{1,t}X_{2,t})^2$	$\|X_{1,t}X_{2,t}\|$	$\sigma^{RM}_{X_{1,t}}$
Bivariate GARCH with constant conditional correlation H_0: $(\omega_{i,0}, \alpha_{i,0}, \beta_{i,0}t)$										
DGP1:(0.4,0.1,0.5)	0.053	0.044	0.049	0.000	0.000	0.000	0.000	0.000	0.000	0.000
DGP2:(0.1,0.1,0.8)	0.086	0.063	0.081	0.000	0.000	0.000	0.000	0.000	0.000	0.000
H_1^A: Break in the dynamics of volatility, $(\beta_{i,j,0},\ \beta_{i,j,1}), i, j = 1, 2$										
DGP1: (0.5, 0.8)	0.999	0.910	0.998	0.622	0.069	0.068	0.792	0.052	0.076	0.128
DGP1: (0.5, 0.1)	0.387	0.751	0.478	0.279	0.014	0.000	0.400	0.022	0.002	0.504
DGP2: (0.8, 0.5)	0.999	0.830	0.889	0.998	0.401	0.263	0.508	1.000	0.422	0.669
H_1^B: Break in the constant of volatility, $(\omega_{i,j,0}, \omega_{i,j,1}), i, j = 1, 2$										
DGP1: (0.4, 0.2)	0.745	0.369	0.466	0.281	0.017	0.001	0.402	0.016	0.002	0.490
DGP2: (0.1, 0.2)	0.812	0.541	0.707	0.058	0.006	0.000	0.097	0.004	0.000	0.036
H_1^C: Break in the correlation coefficient, $(\rho_{12,0}, \rho_{12,1})$										
DGP1: (0.5,0.8)	0.965	0.807	0.933	0.155	0.007	0.000	0.296	0.005	0.004	0.103
DGP1: (0.5,0.3)	0.958	0.652	0.702	0.915	0.085	0.003	0.913	0.094	0.010	0.849
DGP2: (0.5,0.3)	0.961	0.620	0.733	0.890	0.090	0.009	0.925	0.088	0.016	0.407
DGP2: (0.5,0.8)	0.961	0.796	0.908	0.176	0.017	0.003	0.293	0.009	0.003	0.070
Bivariate GARCH with time varying conditional correlation H_1^D: Break in the covariance dynamics, $(\beta_{12,0},\ \beta_{12,1})$										
DGP1: (0.5,0.1)	0.989	0.961	0.995	0.000	0.000	0.000	0.000	0.000	0.000	0.014
DGP2: (0.8,0.4)	1.000	0.967	0.997	0.007	0.050	0.001	0.153	0.005	0.003	0.283

Note: (1) The Kokoszka and Leipus (2000) test statistic is described in Section 3.1. (2) The simulated bivariate GARCH models refer to GARCH-CCC (Constant Conditional Correlation) at (4.2), (4.3), (4.4) and GARCH-VDC (Varying Conditional Correlation) at (4.2), (4.3), (4.5). (3) The models are simulated for 1,000 replications. The superscripts 1 and 0 in the coefficients of each hypothesis in the Table denote the cases with and without change-points, respectively. The entries in DGP(*,*) refer to the coefficients prior to and after the change-point under the alternative hypothesis, as given by the title of each panel. For instance, the entries in DGP1:(0.5,0.8) under the alternative hypothesis H_1^A (in the second panel) denote the constant GARCH coefficients prior to and after the change point. (4) Under the alternative hypotheses H_1^A and H_1^B, the change in parameters refer to both GARCH processes. Under the alternative hypotheses H_1^C and H_1^D, we assess the change in the conditional covariance.

using either $\hat{\sigma}_{RV26,t}$ and $\hat{\sigma}_{RM,t}$. The main result from Table 2 is that the cross-product $Y_{12,t} := X_{1,t}X_{2,t}$ (as opposed to its quadratic and absolute transformations) as well as $\sigma^{RM}_{Y_{12}}$ yield the highest power under the hypotheses of change points in the volatility coefficients (H_1^A and H_1^B) as well as the conditional covariance parameters (H_1^C and H_1^D). It is important to clarify that the normalized returns cross product process $Y_{12,t}$, which involves volatility estimation, has lower power than the true simulated process and has relatively more power in detecting large change points in the context of the GARCH-CCC than GARCH-VDC model.

Table 3. Size, power and frequency distribution of the number of change-points obtained with the Lavielle and Moulines (2000) test when there is a *single break* in a M-GARCH with constant conditional correlation.

Samples, $T = 1000$ and change point, $\pi = 0.5$ and *Segments,* $t_k = 5$												
Normalized returns regression $X\left(\sigma_{i,t}^k\right) = a + bX(\sigma_{j,t}^k) + u_t$												
Volatility Filter, $\sigma_{i,t}^k$	σ_t^{RV26}						σ_t^{RM}					
Lavielle&Moulines	BIC			LWZ			BIC			LWZ		
Number of Breaks	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2
$H_0 : (\omega_{i,0}, \alpha_{i,0}, \beta_{i,0})$												
DGP1:(0.4,0.1,0.5)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**0.98**	0.02	0.00	**1.00**	0.00	0.00
DGP2:(0.1,0.1,0.8)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
H_1^A : Break in the dynamics of volatility with parameters (β_0, β_1)												
DGP1:(0.5, 0.8)	0.00	**1.00**	0.00	0.44	**0.56**	0.00	0.00	**0.98**	0.02	0.38	**0.62**	0.00
DGP1:(0.5, 0.1)	0.70	**0.30**	0.00	1.00	**0.00**	0.00	0.48	**0.52**	0.00	1.00	**0.00**	0.00
DGP2:(0.8, 0.7)	0.02	**0.96**	0.02	0.98	**0.02**	0.00	0.88	**0.12**	0.00	1.00	**0.00**	0.00
DGP2:(0.8, 0.5)	0.02	**0.96**	0.02	0.98	**0.02**	0.00	0.04	**0.94**	0.02	0.94	**0.06**	0.00
H_1^B : Break in the constant of volatility with parameters (ω_0, ω_1)												
DGP1:(0.4, 0.1)	0.04	**0.96**	0.00	0.92	**0.08**	0.00	0.06	**0.94**	0.00	0.94	**0.06**	0.00
DGP1:(0.4, 0.8)	0.10	**0.90**	0.00	1.00	**0.00**	0.00	0.20	**0.80**	0.00	1.00	**0.00**	0.00
DGP2:(0.1, 0.3)	0.10	**0.90**	0.00	1.00	**0.00**	0.00	0.96	**0.04**	0.00	1.00	**0.00**	0.00
H_1^D : Break in the correlation coefficient $(\rho_{12,0}, \rho_{12,1})$												
DGP1:(0.5, 0.3)	0.00	**1.00**	0.00	0.88	**0.12**	0.00	0.00	**1.00**	0.00	0.78	**0.22**	0.00
DGP1:(0.5, 0.8)	0.00	**1.00**	0.00	0.26	**0.74**	0.00	0.00	**0.95**	0.05	0.35	**0.65**	0.00
DGP2:(0.5, 0.3)	0.02	**0.98**	0.00	0.92	**0.08**	0.00	0.00	**0.98**	0.02	0.88	**0.12**	0.00
DGP2:(0.5, 0.8)	0.00	**1.00**	0.00	0.30	**0.70**	0.00	0.88	**0.12**	0.04	1.00	**0.00**	0.00

Notes: (1) The Lavielle and Moulines (2000) test is described in Section 1.2. The Bayesian Information Criterion (BIC) and its modification by Liu, Wu and Zidek (1997) denoted as LWZ are used. The simulations focus on DGP1, DGP2, $T = 1000$ for 500 trials. For comparison purposes the alternative hypotheses of change points are similar to the K&L simulations (Table 2) and extended to larger breaks. Reported is the frequency distribution of the breaks detected. The highlighted numbers refer to the true number of change-points in the simulated process. (2) Explanations for the changes in the coefficients represented by the entries DGP:(*,*) are given in note (3), Table 2. (3) The simulated model is given at (4.2), (4.3) and (4.4).

Table 4. Size, power and frequency distribution of the number of change-points obtained with the Lavielle and Moulines (2000) test when there is a *single break* in a M-GARCH with dynamic conditional covariance.

Samples, $T = 1000$ and change point, $\pi = 0.5$												
Normalized returns regression $X\left(\sigma_{i,t}^k\right) = a + bX\left(\sigma_{j,t}^k\right) + u_t$												
Volatility Filter, $\sigma_{i,t}^k$	σ_t^{RV26}						σ_t^{RM}					
Lavielle & Moulines	BIC			LWZ			BIC			LWZ		
Segments, $t_k = 5$												
Number of Breaks	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2	0	1	≥ 2
$H_0 : (\omega_{i,0}, \alpha_{i,0}, \beta_{i,0})$												
DGP1:(0.4,0.1,0.5)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
DGP2:(0.1,0.1,0.8)	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00	**1.00**	0.00	0.00
H_1^A : Break in the dynamics of volatility with parameters (β_0, β_1)												
DGP1:$(0.5, 0.8)$	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.95	**0.05**	0.00
DGP1:$(0.5, 0.1)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2:$(0.8, 0.7)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP2:$(0.8, 0.5)$	0.54	**0.46**	0.00	1.00	**0.00**	0.00	0.59	**0.41**	0.00	1.00	**0.00**	0.00
H_1^B : Break in the constant of volatility with parameters (ω_0, ω_1)												
DGP1:$(0.4, 0.5)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00	1.00	**0.00**	0.00
DGP1:$(0.4, 0.8)$	0.80	**0.20**	0.00	1.00	**0.00**	0.00	0.44	**0.56**	0.00	1.00	**0.00**	0.00
DGP2:$(0.1, 0.3)$	0.14	**0.86**	0.00	1.00	**0.00**	0.00	0.01	**0.99**	0.00	1.00	**0.00**	0.00
DGP2:$(0.1, 0.2)$	0.96	**0.04**	0.00	1.00	**0.00**	0.00	0.98	**0.02**	0.00	1.00	**0.00**	0.00
H_1^C : Break in the constant of the conditional covariance coefficient $(\omega_{12,0}, \omega_{12,1})$												
DGP1:$(0.4, 0.1)$	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.26	**0.74**	0.00
DGP1:$(0.4, 0.8)$	0.80	**0.20**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.22	**0.78**	0.00
DGP2:$(0.1, 0.3)$	0.10	**0.90**	0.00	1.00	**0.00**	0.00	0.96	**0.04**	0.00	1.00	**0.00**	0.00
H_1^D : Break in the dynamics of the conditional covariance coefficient $(b_{12,0}, b_{12,1})$												
DGP1:$(0.5, 0.8)$	0.00	**1.00**	0.00	1.00	**0.00**	0.00	0.00	**1.00**	0.00	0.94	**0.06**	0.00
DGP1:$(0.5, 0.1)$	1.00	**0.00**	0.00	1.00	**0.00**	0.00	0.42	**0.58**	0.00	1.00	**0.00**	0.00
DGP2:$(0.8, 0.5)$	0.00	**1.00**	0.00	0.92	**0.08**	0.00	0.00	**1.00**	0.00	0.66	**0.34**	0.00

Notes: As in the notes (1) and (3) of Table 3, the simulated model is given at (4.2), (4.3) and (4.5).

The change-point hypothesis in multivariate conditional volatility models is also examined using the Lavielle and Moulines (L&M) test. Table 3 shows the L&M least squares regression test results for pairs of normalized returns $X_{1,t} = \theta_{12}' + \eta_{12}' X_{2,t} + v_{12,t}$, in the context of the M-GARCH-CCC. The highlighted results show that the BIC yields more power than the LWZ criterion for the L&M test which detects breaks in both directions and DGPs, except when those are small in size (e.g., a 0.1 parameter change). The results regarding the remaining alternative hypotheses (H_1^A and H_1^B) show that the L&M test also detects breaks in the bivariate relationship of normalized returns when the source of these change-points rests in the univariate GARCH dynamics as well as breaks in the co-movements (H_1^C). The above results also hold if the simulated process is an M-GARCH-VDC as shown in Table 4, except that the size of the change-point needs to be even larger in either the conditional variance or covariance dynamics for the test to exhibit power. It is also interesting to note that in comparing the normalizing volatility filters we find that the regression involving $X_{RM,t}$ yields more power in detecting change-points in the conditional covariance of the M-GARCH-VDC, whereas for the M-GARCH-CCC both $X_{RM,t}$ and $X_{RV26,t}$ yield similar power properties.

5. Empirical Analysis

5.1. Co-movements of FX normalized returns

The empirical section of the paper investigates the bivariate relationship between the daily YN/US$ and DM/US$ risk adjusted returns over a decade, and tests for structural breaks in their co-movements. The empirical results complement the Monte Carlo analysis by examining further the stochastic properties of normalized FX returns and investigating the presence of structural breaks. The discussion is organized as follows. First, we test the hypotheses of normality and independence for all YN/US$ and DM/US$ standardized returns as well as the statistical adequacy of their regression representation. Second, we examine the stability of this bivariate relationship by testing for change-points using the Kokoszka and Leipus (2000), Horváth (1997) and Lavielle and Moulines (2000) tests, which are valid for heavy tailed as well as weakly and strongly dependent processes. The timing and numbers of breaks are also estimated. The data source is Olsen and Associates. The original sample for a decade, from December 1st, 1986 to November 30th, 1996 is 1,052,064 five-minute return observations (2,653 days $\cdot$ 288 five-minute intervals per day). The returns for some days were removed from the sample to avoid regular and predictable market closures which affect the characterization of the volatility dynamics. A description of the data removed is found in Andersen, Bollerslev, Diebold and Labys (2001). The final sample includes 705,024 five-minute returns reflecting 2,448 trading days.

The statistical properties of daily returns normalized by a number of volatility filters are examined for the two FX series. First we focus on the temporal dependence and distributional properties of normalized returns. It is well documented that daily asset returns are characterized by a martingale difference with second-order temporal dynamics and a distribution that exhibits heavy-tails. Therefore it would be interesting to examine whether these purely data-driven volatility filters also adequately capture the second-order dynamics of asset returns. This is examined by testing the hypothesis of remaining linear and second-order temporal dependence effects in normalized returns. Detailed empirical results do not reject the hypothesis that these normalized returns are approximately independent, see Andreou and Ghysels (2002c, Tables 5 and 6). Specifically, it is found that the efficiency depends on the sampling frequency, window length and estimation method. The combination of rolling estimation and optimal window produces nearly independent standardized FX returns series. Second, temporal aggregation of intraday returns requires a longer lag of volatility so as to capture the dependence in normalized returns, and the empirical findings support the continuous record asymptotics of the efficiency of volatility filters. The distributional properties of normalized returns are assessed in Table 5 for the YN/US$ and DM/US$. Both the Jarque and Bera (1980) and Anderson and Darling (1954) tests result provide no empirical support of the normality hypothesis (at the 10% significance level) for any of the daily standardized returns series, mainly due to excess kurtosis in both the spot volatility (SV) normalized returns, $X_{SV,t}$, as well as the $X_{(H)QV,t}$ series. The exception to this result is $X_{QV1,t}$, which appears to support the normality hypothesis at the 5-minute sampling frequency. Nevertheless at the lower sampling frequencies $X_{QV1,t}$ is also non-normal. At the 5-minute sampling frequency the sample skewness and kurtosis coefficients suggest that the empirical distributions for all standardized returns are leptokurtic except for $X_{QV1,t}$, which actually appears to be platykurtic with sample kurtosis coefficient below 3 for all intraday frequencies. The empirical results match the simulation evidence in Table 1. Moreover, it is interesting to note that a window length beyond one day in QV filters, as well as rolling instead of block sampling estimation methods, yield excess kurtosis in the empirical distribution. It is worth noting that the daily and most intraday volatility filters result in non-normality due to both excess kurtosis and, in most cases, asymmetry. This result may be due to an underlying non-normal distribution and/or the presence of jumps and breaks in the risk adjusted returns process. The empirical tail behavior implied by $X_{QV,t}$ and $X_{HQV,t}$ differ and the latter are found to be relatively more leptokurtic. Moreover, as the window length increases for both QV and SV filters, the distribution of the respective standardized returns becomes more leptokurtic.

The above results suggest that the ratio transformation of daily returns-to-volatility based on data-driven volatility filters can yield a process with a relatively simple statistical structure. Hence we proceed to examine the multivariate relationship of normalized returns in a regression context. The dynamic structure of risk adjusted returns using Granger causality tests and the existence of a linear regression relationship for YN/US$ and DM/US$ normalized returns can also be found in Andreou and Ghysels (2002c, Table 6). There is no significant empirical evidence of a lead-lag relationship between the co-movements of the two FX series. The simple linear OLS regression results of the two normalized FX returns are reported in Table 6 for the 5- and 30-minute frequencies. In all cases the estimated regression coefficient is highly significant and ranges from 0.6 to 0.75 as representing the contemporaneous covariance structure of standardized returns in the DM and YN vis-à-vis the US$. The statistical adequacy of this regression relationship is examined. All regression results for $X_{SV,t}$ and $X_{(H)QV,t}$ support the independence hypothesis (except $X_{QV1,t}$ in the 30-minute sampling frequency). Similarly, the empirical results show that the static regressions exhibit non-normal conditional distribution for the two FX risk adjusted returns. These results open the route for regression type techniques in detecting change-points and suggest that the empirical conditional covariance process does not exhibit significant dynamics.

Table 5. Normality test results for daily YN/US$ standardized returns based on various intraday sampling frequencies.

	YN/US$						DM/US$					
	5min. frequency			30min. frequency			5min. frequency			30min. frequency		
	Sk.	AD	BJ	Sk.	AD	BJ	Sk.	AD	BJ	Sk	AD	BJ
	Kr.	p-value	p-value	Kr.	p-value	p-value	Kr.	p-value	p-value	Kr.	p-value	p-value
X(RM)	-0.215	4.305	51.511	-0.174	9.062	167.08	-0.012	1.890	8.210	0.142	7.589	170.59
	3.585	(0.000)	(0.000)	4.260	(0.000)	(0.000)	3.289	(0.000)	(0.017)	4.290	(0.000)	(0.000)
X(RV26)	-0.251	7.566	148.74	-0.226	15.403	446.74	-0.019	4.233	55.713	0.256	12.430	451.08
	4.127	(0.000)	(0.000)	5.089	(0.000)	(0.000)	3.754	(0.000)	(0.000)	5.086	(0.000)	(0.000)
X(RV52)	-0.309	11.196	327.21	-0.380	25.022	1471.9	-0.030	6.788	132.50	0.277	19.598	1359.3
	4.722	(0.000)	(0.000)	6.805	(0.000)	(0.000)	3.989	(0.000)	(0.000)	6.688	(0.000)	(0.000)
X(QV1)	-0.030	0.558	1.064	-0.055	1.029	10.407	-0.011	0.418	6.605	-0.012	1.214	17.845
	2.915	(0.149)	(0.588)	2.693	(0.010)	(0.000)	2.741	(0.328)	(0.037)	2.573	(0.004)	(0.000)
X(QVk)	-0.091	2.720	35.943	-0.093	1.384	12.914	-0.005	0.880	2.479	-0.004	0.491	0.256
	3.579	(0.000)	(0.000)	3.312	(0.001)	(0.000)	3.159	(0.024)	(0.289)	3.051	(0.219)	(0.880)
X(QVl)	-0.113	5.598	105.5	-0.192	7.459	151.9	-0.021	1.945	3.292	0.009	3.215	3.699
	3.992	(0.000)	(0.000)	4.193	(0.000)	(0.000)	3.359	(0.000)	(0.001)	3.194	(0.000)	(0.157)
X(HQV1)	-0.138	5.248	120.04	-0.134	3.355	59.811	-0.110	2.942	132.12	-0.092	2.676	82.549
	4.073	(0.000)	(0.000)	3.736	(0.000)	(0.000)	4.142	(0.000)	(0.000)	3.902	(0.000)	(0.000)
X(HQVk)	-0.191	8.683	245.61	-0.149	9.976	314.04	-0.082	4.649	151.52	-0.059	5.664	132.39
	4.539	(0.000)	(0.000)	4.769	(0.000)	(0.000)	4.235	(0.000)	(0.000)	4.159	(0.000)	(0.000)
X(HQVl)	-0.202	10.719	327.82	-0.179	11.298	380.72	-0.054	5.555	154.06	-0.099	6.671	280.06
	4.787	(0.000)	(0.000)	4.943	(0.000)	(0.000)	4.251	(0.000)	(0.000)	4.683	(0.000)	(0.000)

Note: The volatility filters are defined in Section 2.2. The data set refers to the 5-minute YN/US$ from 1/12/86 to 30/11/96 which yields a daily sample size of T=2446 days and is adjusted for a subsample of 2346, excluding the first 100 observations as a result of the rolling volatility estimators. The window lengths are k=2,4,6 and l=3,8,12 days for the 5-, 30- and 60-minutes frequency, respectively. The sample skewness and kurtosis (Sk and Kr., respectively) are reported. Statistics reported refer to p-values, the Anderson-Darling (AD) and Bera-Jarque (BJ) tests.

Table 6. Linear regression results of daily YN/US$ on DM/US$ standardized returns based on intra-day sampling frequencies.

	5-minute sampling frequency							
	OLS results		Residual Misspecification results					
	const. p-value	beta p-value	BJ p-value	Sk. Kr.	ARCH(1) p-value	ARCH(5) p-value	LM(1) p-value	LM(5) p-value
X(RM)	-0.017	0.603	601.95	-0.566	2.468	1.115	1.220	0.702
	(0.276)	(0.000)	(0.000)	5.209	(0.116)	(0.350)	(0.269)	(0.622)
X(RV26)	-0.021	0.604	884.21	-0.597	1.847	1.091	1.298	0.917
	(0.208)	(0.000)	(0.000)	5.760	(0.174)	(0.363)	(0.225)	(0.469)
X(RV52)	-0.023	0.603	1542.4	-0.766	4.217	1.987	1.619	0.729
	(0.172)	(0.000)	(0.000)	6.664	(0.040)	(0.078)	(0.203)	(0.601)
X(QV1)	0.004	0.605	54.153	-0.223	1.508	3.238	0.394	0.440
	(0.759)	(0.000)	(0.000)	3.595	(0.219)	(0.006)	(0.530)	(0.821)
X(QV2)	-0.004	0.607	284.72	-0.400	0.507	1.524	1.603	1.524
	(0.784)	(0.000)	(0.000)	4.507	(0.476)	(0.179)	(0.206)	(0.179)
X(QV3)	-0.003	0.609	283.44	-0.397	0.513	1.538	1.540	0.588
	(0.821)	(0.000)	(0.000)	4.505	(0.474)	(0.175)	(0.215)	(0.709)
X(HQV1)	-0.0002	0.607	442.81	-0.422	0.069	0.959	2.335	0.599
	(0.861)	(0.000)	(0.000)	4.902	(0.793)	(0.442)	(0.127)	(0.701)
X(HQV2)	-0.0003	0.603	1117.7	-0.614	0.174	0.675	2.418	0.607
	(0.611)	(0.000)	(0.000)	6.152	(0.676)	(0.643)	(0.120)	(0.694)
X(HQV3)	-0.0003	0.602	1435.1	-0.662	0.420	0.679	2.274	0.598
	(0.530)	(0.000)	(0.000)	6.597	(0.517)	(0.639)	(0.132)	(0.702)

	30-minute sampling frequency							
	OLS results		Residual Misspecification results					
	const. p-value	beta p-value	BJ p-value	Sk. Kr.	ARCH(1) p-value	ARCH(5) p-value	LM(1) p-value	LM(5) p-value
X(RM)	-0.032	0.746	52786	-2.068	0.010	0.073	1.749	2.569
	(0.011)	(0.000)	(0.000)	25.862	(0.919)	(0.996)	(0.186)	(0.025)
X(RV26)	-0.032	0.743	84087	2.463	0.022	0.044	0.854	2.345
	(0.024)	(0.000)	(0.000)	31.907	(0.883)	(0.999)	(0.355)	(0.039)
X(RV52)	-0.038	0.722	175997	-3.336	1.229	0.039	1.229	2.051
	(0.009)	(0.000)	(0.000)	44.895	(0.268)	(0.999)	(0.268)	(0.069)
X(QV1)	0.007	0.600	31.273	-0.193	0.786	3.492	0.180	0.459
	(0.659)	(0.000)	(0.000)	3.414	(0.375)	(0.004)	(0.671)	(0.807)
X(QV2)	0.0006	0.607	183.84	-0.329	0.475	1.789	1.281	0.523
	(0.971)	(0.000)	(0.000)	4.204	(0.491)	(0.112)	(0.258)	(0.759)
X(QV3)	-0.016	0.618	609.2	-0.485	1.535	0.350	1.028	0.499
	(0.016)	(0.016)	(0.000)	5.244	(0.215)	(0.882)	(0.311)	(0.777)
X(HQV1)	0.0002	0.605	201.57	-0.325	0.031	1.208	1.465	0.352
	(0.938)	(0.000)	(0.000)	4.282	(0.861)	(0.303)	(0.226)	(0.881)
X(HQV2)	-0.0006	0.632	803.44	-0.514	1.223	0.716	1.679	0.485
	(0.648)	(0.000)	(0.000)	5.681	(0.269)	(0.612)	(0.195)	(0.788)
X(HQV3)	-0.0007	0.609	1187.5	-0.572	0.777	0.196	1.618	0.474
	(0.407)	(0.000)	(0.000)	6.297	(0.574)	(0.964)	(0.204)	(0.796)

Note: The notes in Tables IV, VI and VIII apply.

5.2. Empirical evidence for breaks in FX co-movements

The above empirical regularities of the DM/US$ and YN/US$ normalized returns satisfy the conditions of the least squares methods in Bai and Perron (1998) and Lavielle and Moulines (2000), as well as the CUSUM test of Horváth (1997) and of Kokoszka and Leipus (1998, 2000) discussed in Section 3.

The K&L change-point test results for the conditional covariance between the DM/US$ and YN/US$ are reported in Table 7. The results show that the univariate normalized returns (using any volatility filter transformation) appear to be time-homogeneous processes. However, for the cross-product of the two FX normalized returns, the K&L test shows that there is strong evidence of a change-point in their co-movements. The breaks are detected in all specifications of normalized returns and they occur at the same point in time, namely at March 23th, 1995 at which the statistic first exceeds the 5% control limit. This event is related to a period of high uncertainty and a series of bilateral interventions by the Bank of Japan and the Fed (see for instance the Asian Wall Street Journal). It is worth mentioning the parametric CUSUMSQ test (Brown, Durbin and Evans (1975)) also presents empirical evidence for the instability in the linear regression of the two FX risk adjusted returns. However, we emphasize that these results are based on the statistical adequacy of the normal linear regression model. The presence of heavy tailed distributions in normalized returns (or generally deviations from normality) requires more efficient statistical inference methods for testing the existence of breaks. Similarly, although the parametric CUSUM is robust to deviations from normality, this result does not extend to the CUSUM of squares (Ploberger and Kramer (1986)). It is worth mentioning that an application of the parametric CUSUM does not detect any change-points.

These results are complemented by testing for multiple breaks using the L&M regression method and the two information criteria, BIC and LWZ, also reported in Table 7. Given the empirical results in the previous section which support a static regression framework for the two FX normalized returns, we apply the L&M test in the context of (3.10). The number and timing of breaks detected (reported in Table 7) not only vary depending on the information criterion but also on the specification of normalized returns. The general result is that the tests choose between zero, one and two change-points, and the break dates are relatively more consistent for $X_{(H)QV,t}$ using both criteria. This is also related to the empirical results comparing the different normalizations. The two change-points detected are associated with the events of the US stock market crash in October 1987, and the period before the repeated bilateral FX market interventions in March 1995. From the simulation results we learn that the BIC criterion is relatively more powerful and this is complemented by the empirical evidence, which in most cases detects two change-points. Concluding we find that the co-movements in YN/US$ and DM/US$ normalized returns for the most efficient class of filters present evidence for change-points using the recent CUSUM and least-squares methods in K&L and L&M, respectively. Both approaches yield consistent results about the change-points in the co-movements, whereas the latter procedure complements the former by detecting an additional break in the sample.

Table 7. Change-point test results of daily YN/US$ on DM/US$ standardized returns based on 30 minute intra-day sampling frequency.

	Kokoszka and Leipus Change-point Test			
	Normalized Returns		Comovements	Break Dates
	YN(σ_t)	DM(σ_t)	YN(σ_t)*DM(σ_t)	k^*
	$\frac{U_{\max}}{\hat{\sigma}_{VARHAC}}$	$\frac{U_{\max}}{\hat{\sigma}_{VARHAC}}$	$\frac{U_{\max}}{\hat{\sigma}_{VARHAC}}$	
X(RM)	0.706	0.839	5.215*	Mar.95
X(RV26)	0.810	0.788	1.413*	Oct.87
X(RV52)	0.806	0.856	1.178	-
X(QV1)	1.106	0.937	3.503*	Oct.87
X(QV4)	1.133	0.929	2.980*	Oct.87
X(QV8)	1.184	0.914	2.245*	Oct.87
X(HQV1)	1.086	0.879	2.453*	Oct.87
X(HQV4)	1.128	1.003	1.984*	Oct.87
X(HQV8)	1.149	0.945	1.818*	Oct.87

Lavielle and Moulines Multiple Breaks Test							
Normalized Returns				Comovements		Break Dates	
YN(σ_t)		DM(σ_t)		YN$(\sigma_t) = a + b$DM$(\sigma_t) + u_t$		k^*	
SIC(k)	LWZ(k)	SIC(k)	LWZ(k)	SIC(k)	LWZ(k)	SIC(k)	LWZ(k)
-0.042(0)	-0.041(0)	-0.028(0)	-0.027(0)	-0.298(1)-0.301(2)	-0.285(1)-0.184(0)	Oct.87,Mar.95	Mar.95
-0.014(0)	-0.013(0)	-0.004(0)	-0.004(0)	-0.497(1)-0.496(0)	0.495(0)	Oct.87	-
0.037(0)	0.037(0)	0.032 (0)	0.033(0)	-0.438(2)-0.437(1)	-0.435(0)	Oct.87,Mar.95	-
-0.067(0)	-0.066(0)	-0.004(0)	-0.004(0)	-0.529(2)-0.528(1)	-0.515(1)-0.512(0)	Oct.87,Mar.95	Oct.87
0.015(0)	0.015(0)	0.066 (0)	0.066(0)	-0.469(2)-0.467(1)	-0.454(1)-0.452(0)	Oct.87,Mar.95	Oct.87
0.060(0)	0.060(0)	0.078 (0)	0.079(0)	-0.438(2)-0.435(1)	-0.426(0)	Oct.87,Mar.95	-
-3.684(0)	-3.684(0)	-3.766(0)	-3.765(0)	-4.309(1)-4.286(0)	-4.295(1)-4.285(0)	Oct.87	Oct.87
-5.085(0)	-5.085(0)	-5.110(0)	-5.109(0)	-5.629(2)-5.628(1)	-5.614(1)-5.611(0)	Oct.87,Mar.95	Oct.87,Mar.95
-5.803(0)	-5.802(0)	-5.827(0)	-5.827(0)	-6.337(2)-6.336(1)	-6.323(2)-6.321(1)	Oct.87,Mar.95	Oct.87,Mar.95

Note: The break dates of returns standardized by the class of quadratic variation filters X((H)QV) results in more consistent results. Hence we focus our discussion on these specifications.

6. Conclusions

We have proposed reduced form procedures designed to uncover breaks in the co-movements of financial markets via testing for change-points in linear relationships involving returns normalized by conditional volatility. There are several advantages to using normalized returns. Among the advantages we note that (1) the covariance of normalized returns capture conditional correlations, (2) they reduce the complexity of multivariate volatility models along the same lines as Engle (2002), Engle and Sheppard (2002) and Tse and Tsui (2002), (3) they enable us to adopt two-stage procedure consisting of a purely data-driven nonparametric first stage and a semiparametric second stage. Though our procedures share some features with the two-stage estimation procedure of DCC models, we take a reduced form view that suffices for the change-point test purpose. Since the parametric structure of the volatility co-movements are largely left unspecified we cover a larger class of multivariate specifications, including factor ARCH models. Another main advantage of employing the two-step procedure is that the statistical inference methods allow for departures from normality and therefore are robust to heavy tailed distributions. It should also be noted that the returns-to-volatility process relates to various measures of portfolio performance. Such measures include the Treynor ratio which is the square of the Sharpe ratio (Treynor and Black (1973)). Our two-stage procedure also applies to various alternative functional forms of normalized returns. Hence, we can examine structural breaks in Treynor-Black and other measures, and again not require normality assumptions to do so (similar to the Jobson and Korkie (1980,1981) approach for the normal case).

We have restricted the simulation and empirical investigations to bivariate models. Extensions to the multidimensional vector of n assets are routes for further research. The methods proposed can be adapted to examine the $n-$homogeneity of the conditional correlation of the cross-section of assets when n is large in the context of M-GARCH-CCC models, in a similar way to Horváth, Kokoszka and Steinebach (1999) for the mean of n-dependent observations. In addition, the nonparametric testing approach presented here can be complemented with parametric methods for identifying the different sources of structural change in the variance-covariance dynamics. Further research in a system of conditional covariance equations for testing change-points is a useful extension of the present analysis.

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