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SOME NEW THREE-LEVEL ORTHOGONAL MAIN EFFECTS PLANS ROBUST TO MODEL UNCERTAINTY

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Abstract: In this paper, we list some new orthogonal main effects plans for three-level designs for 4, 5 and 6 factors in 18 runs and compare them with designs obtained from the existing L_{18} orthogonal array. We show that these new designs have better projection properties and can provide better parameter estimates for a range of possible models. Additionally, we study designs in other smaller run-sizes when there are insufficient resources to perform an 18-run experiment. Plans for three-level designs for 4, 5 and 6 factors in 13 to 17 runs are given. We show that the best designs here are efficient and deserve strong consideration in many practical situations.

Key words and phrases: Factor screening, fold-over designs, fractional replicate, orthogonal array, projection.

1. Introduction

Many orthogonal main effects designs for three-level factorial structures are well known, are described in books such as that by Wu and Hamada (2000), and have been used extensively in practice. It is usually assumed that identification of factors with large main effects is the sole purpose of the experiment. Motivated by Hamada and Wu (1992), Tsai, Gilmour and Mead (2000) used a two-stage analysis strategy that considers interactions as well as main effects for the analysis of screening experiments with complex aliasing patterns. Under this analysis strategy, it is important to construct designs for screening experiments that are efficient not only for identifying the active factors, but also for fitting a range of possible models that can contain some interactions as well as main effects. For constructing designs with good projection properties, Tsai et al. (2000) defined and used a columnwise procedure to generate three-level designs for six factors in 18 runs, all of which are equally efficient for fitting the six-factor main effects model and any sub-model of it. They also introduced a criterion, denoted by $Q(\Gamma^{(k)})$ for a k-factor design, which averages an approximation to A_s-efficiency over lower-dimensional projections of the design. This criterion is used to explore the projection efficiencies of the design. Designs with lower $Q(\Gamma^{(k)})$ are more likely to have efficient projections and, on average, can provide better parameter estimates over a range of models than designs with higher $Q(\Gamma^{(k)})$.

One of the frequently used three-level orthogonal arrays in industrial experiments is the $L_{18}(3^7)$, an 18-run design for seven three-level factors. Wang and Wu (1995) and Cheng and Wu (2001) studied the projection properties of the $L_{18}(3^7)$ orthogonal array when projected onto sets of three and four factors. Cheng and Ye (2004) permuted the levels of the projected designs to generate some additional 18-run designs for three and four factors. However, the designs studied in these papers always project onto designs for three factors that are formed by putting two regular 3^{3-1} factorials together and have either 18, 15 or 9 distinct points. In this paper, we show that by using a design procedure that has no restriction on designs for their projections onto three factors, more new designs can be generated. All these designs are equally efficient for fitting the main effects model. Additionally, the best designs we have found have better projection properties and can provide better parameter estimates for a range of possible models. These designs deserve strong consideration in many practical situations.

The purpose of this paper is to compare designs generated by the criterion of Tsai et al. (2000) with those obtained by Cheng and Ye (2004) and those obtained from the L_{18} orthogonal array. In Section 2, we give some new orthogonal main effects three-level designs for 4 to 6 factors in 18 runs and compare them with those obtained from the L_{18}. Additionally, we consider the situation when there are insufficient resources to perform an 18-run experiment. In Section 3, we list some of the best designs for 4 to 6 factors in 13 to 17 runs, and compare them with the D-optimal main effects designs obtained from the SAS procedure OPTEX (SAS Institute, 1995, Pt.6). In Section 4 we summarise the conclusions.

2. Three-Level Designs in 18 Runs

2.1. Designs for three factors in 18 runs

The general procedure to construct regular three-level factorials is through defining contrast subgroups. Consider an example of a regular 3^{3-1} factorial. The nine level-combinations of a regular 3^{3-1} factorial are determined by a defining contrast relation such that $x_1 + \alpha_2 x_2 + \alpha_3 x_3 = a \pmod 3$, where $\alpha_i = 1$ or 2, $a = 0, 1$ or 2 and the levels of a factor, x_i, are coded as 0, 1 and 2. It is easily verified that there are twelve sets of α's and a's corresponding to the twelve regular 3^{3-1} factorials. Each of them can be represented by a 3×3 Latin square.

These twelve designs are *combinatorially isomorphic* since a design can be obtained from any other by permutations of rows, columns and levels of factors. However, when we are interested in checking the linearity of the response, we use the linear and quadratic contrasts, with coefficients $(-1, 0, 1)$ and $(1/2, -1, 1/2)$

respectively, to decompose the main effects of factors. Designs obtained from one another by some relabellings of levels of factors, namely switching 0 with $+1$ or 0 with -1, are said to be in different *design families* as defined by Tsai et al. (2000), or *geometrically non-isomorphic* as defined by Cheng and Wu (2001). For this classification, the twelve regular 3^{3-1} designs are divided into two design families: one consists of Latin squares with a center point (0,0,0); the other consists of Latin squares without a center point. Only one design in a design family is kept for representing designs in that design family.

By using the design procedure of Tsai et al. (2000), we generate the 13 three-level designs for three factors in 18 runs shown in Table 1. All of these 13 designs are equally efficient for fitting the main effects model assuming no interactions exist. Eight of these designs can be obtained by putting two Latin squares together and have either 18, 15 or 9 distinct points. Designs $D(3)$ and $D(5)$ have 18 points, $D(1)$, $D(6)$, $D(8)$ and $D(11)$ have 15 points, and $D(12)$ and $D(13)$ have nine repeated points.

Table 1. Plans for designs for three factors in 18 runs.

D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13
− − −	− − −	− − −	− − −	− − −	− − −	− − −	− − −	− − −	− − −	− − −	− − −	− − −
− − +	− − 0	− − +	− − 0	− − 0	− − 0	− − 0	− − −	− − −	− − −	− − −	− − −	− − −
− 0 −	− 0 −	− 0 −	− 0 −	− 0 −	− 0 −	− 0 −	− 0 0	− 0 0	− 0 0	− 0 0	− 0 +	− 0 0
− 0 0	− 0 +	− 0 0	− 0 +	− 0 +	− 0 +	− 0 +	− 0 +	− 0 +	− 0 +	− 0 0	− 0 +	− 0 0
− + 0	− + 0	− + 0	− + 0	− + 0	− + 0	− + 0	− + 0	− + 0	− + 0	− + +	− + 0	− + +
− + +	− + +	− + +	− + +	− + +	− + +	− + +	− + +	− + +	− + +	− + +	− + 0	− + +
0 − 0	0 − 0	0 − 0	0 − +	0 − −	0 − +	0 − −	0 − +	0 − 0	0 − 0	0 − 0	0 − +	0 − +
0 − 0	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +	0 − +
0 0 +	0 0 −	0 0 −	0 0 −	0 0 0	0 0 0	0 0 0	0 0 −	0 0 −	0 0 −	0 0 +	0 0 0	0 0 −
0 0 +	0 0 +	0 0 +	0 0 0	0 0 +	0 0 0	0 0 0	0 0 0	0 0 +	0 0 0	0 0 −	0 0 0	0 0 −
0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + −	0 + 0
0 + −	0 + 0	0 + 0	0 + 0	0 + 0	0 + −	0 + +	0 + 0	0 + 0	0 + +	0 + 0	0 + −	0 + 0
+ − −	+ − −	+ − −	+ − −	+ − 0	+ − −	+ − 0	+ − 0	+ − 0	+ − 0	+ − 0	+ − 0	+ − 0
+ − +	+ − +	+ − 0	+ − 0	+ − +	+ − 0	+ − +	+ − 0	+ − +	+ − +	+ − +	+ − 0	+ − 0
+ 0 −	+ 0 0	+ 0 0	+ 0 0	+ 0 −	+ 0 −	+ 0 −	+ 0 −	+ 0 −	+ 0 −	+ 0 −	+ 0 −	+ 0 +
+ 0 0	+ 0 0	+ 0 +	+ 0 +	+ 0 0	+ 0 +	+ 0 +	+ 0 +	+ 0 0	+ 0 +	+ 0 +	+ 0 −	+ 0 +
+ + 0	+ + −	+ + −	+ + −	+ + −	+ + 0	+ + −	+ + −	+ + −	+ + −	+ + −	+ + +	+ + −
+ + +	+ + +	+ + +	+ + +	+ + +	+ + +	+ + 0	+ + +	+ + +	+ + 0	+ + 0	+ + +	+ + −

For these designs, the linear and quadratic effects of a factor are sometimes correlated with interactions not involving that factor, and the interactions are correlated with each other. These designs have different efficiencies for fitting the models that contain some interactions as well as main effects. Table 2 gives values of the $Q(\Gamma^{(3)})$ criterion and the A_s criterion (excluding the intercept) for the second-order model for these designs. Note that $Q(\Gamma^{(3)})$ is an average of an approximate A_s criterion, the average being over the 94 models that contain some or all interactions and main effects of the three factors. (See Tsai et al. (2000) for more details. Note that the $Q(\Gamma^{(3)})$ values for $D(12)$ and $D(13)$ in Table 1 of Tsai et al. (2000) are misprinted.) This table shows that designs that are not formed by putting two Latin squares together, those labelled $D(2)$, $D(4)$, $D(7)$, $D(9)$ and $D(10)$ in Table 1, sometimes have more efficient parameter estimates than those obtained from the L_{18}, even for the second-order model. Notice that $D(12)$ and $D(13)$ are sets of nine duplicated points. There are not enough degrees of freedom to obtain unique estimates for all the effects of the second-order model (with ten parameters). Cheng and Wu (2001) studied the projected three-factor designs from the L_{18} array and did not separate $D(12)$ and $D(13)$.

Table 2. Properties of designs for three factors in 18 runs.

Design	$Q(\Gamma^{(3)})$	A_s	Design	$Q(\Gamma^{(3)})$	A_s
1	0.5148	0.1121	8	0.5635	0.1741
2	0.5236	0.1158	9	0.5656	0.1529
3	0.5301	0.1334	10	0.5683	0.1394
4	0.5304	0.1289	11	0.5946	0.1688
5	0.5328	0.1187	12	0.5993	∞
6	0.5378	0.1420	13	0.6099	∞
7	0.5426	0.1204			

2.2. Designs for four factors in 18 runs

As done for designs for three factors in 18 runs, new four-factor designs are generated by the design procedure that does not force designs to have three-factor projected designs formed by putting two Latin squares together, and 129 designs are obtained in total. Table 3 lists some of the orthogonal main effects designs for four factors, in which designs 5 and 12 are the two designs from the projections of the L_{18} orthogonal array. Table 4 gives the projection properties and the values of $Q(\Gamma^{(4)})$ and A_s for the second-order model for some four-factor designs. It shows that the best designs with lower values of $Q(\Gamma^{(4)})$ are more likely to project onto better designs for three factors. Among the designs we have found, design 1 is the most efficient for fitting the second-order model. It does not always project onto designs for three factors that are formed by putting two Latin squares together. Note that the boldface for the projected designs in Table 4 represents the three-factor designs that are formed by putting two Latin squares together.

Table 3. Plans for designs for four factors in 18 runs.

Design 1	Design 2	Design 5	Design 12
$- - - -$	$- - - -$	$- - - -$	$- - - -$
$- - + 0$	$- - 0\ 0$	$- - + 0$	$- - + 0$
$- 0 - +$	$- 0 - 0$	$- 0 - +$	$- 0 - +$
$- 0\ 0\ 0$	$- 0 + +$	$- 0\ 0\ 0$	$- 0\ 0 -$
$- + 0 -$	$- + 0 +$	$- + 0 -$	$- + 0\ 0$
$- + + +$	$- + + -$	$- + + +$	$- + + +$
$0 - 0 -$	$0 - 0 +$	$0 - 0 -$	$0 - 0\ 0$
$0 - 0 +$	$0 - + -$	$0 - 0 +$	$0 - + +$
$0\ 0 + -$	$0\ 0 - +$	$0\ 0 + -$	$0\ 0 - 0$
$0\ 0 + +$	$0\ 0 + 0$	$0\ 0 + 0$	$0\ 0 + -$
$0 + - 0$	$0 + - -$	$0 + - 0$	$0 + - -$
$0 + - 0$	$0 + 0\ 0$	$0 + - +$	$0 + 0 +$
$+ - - +$	$+ - - 0$	$+ - - 0$	$+ - - +$
$+ - + 0$	$+ - + +$	$+ - + +$	$+ - 0 -$
$+ 0 - -$	$+ 0\ 0 -$	$+ 0 - -$	$+ 0\ 0 +$
$+ 0\ 0\ 0$	$+ 0\ 0 -$	$+ 0\ 0 +$	$+ 0 + 0$
$+ + 0 +$	$+ + - +$	$+ + 0\ 0$	$+ + - 0$
$+ + + -$	$+ + + 0$	$+ + + -$	$+ + + -$

Table 4. Properties of designs for four factors in 18 runs.

Design	$Q(\Gamma^{(4)})$	A_s	Three factors projections {123, 124, 134, 234}
1	0.9276	0.1585	**1** 2 2 **6**
2	0.9375	0.1860	2 4 4 2
3	0.9460	0.1797	2 **6** 2 **3**
5	0.9493	0.1746	**1 5 3 5**
12	0.9587	0.2139	**3 5 5 3**
128	1.2832	∞	**12 12 12 12**
129	1.3214	∞	**13 13 13 13**

If we restrict our attention to four-factor designs that project onto designs for three factors that are formed by putting two Latin squares together, we find that there are 36 such designs for four factors. Cheng and Wu (2001) and Cheng and Ye (2004) studied only the four-factor designs obtained from the projections of the L_{18} array and reported 21 four-factor projected designs.

2.3. Designs for five factors in 18 runs

Considering the projected five-factor designs from the existing L_{18}, there are six different designs. By the $Q(\Gamma^{(5)})$ criterion, they rank 128, 129, 171, 186, 253 and 292 among the 320 five-factor designs that we generated. Table 5 lists plans for the best three designs for five factors in 18 runs, and design 28 that is the best design for which all its 10 sets of three-factor projections can be formed by putting two Latin squares together.

Table 5. Plans for designs for five factors in 18 runs.

Design 1	Design 2	Design 3	Design 28
$- - - - 0$	$- - - - -$	$- - - - 0$	$- - - - -$
$- - 0 + 0$	$- - + 0\ 0$	$- - 0\ 0 -$	$- - + 0 +$
$- 0 - 0 -$	$- 0 - + +$	$- 0 - + +$	$- 0 - + +$
$- 0 + - +$	$- 0\ 0 - 0$	$- 0 + + -$	$- 0\ 0 - 0$
$- + 0\ 0 +$	$- + 0 + -$	$- + 0 - +$	$- + 0\ 0 -$
$- + + + -$	$- + + 0 +$	$- + + 0\ 0$	$- + + + 0$
$0 - 0\ 0 +$	$0 - 0\ 0 -$	$0 - + - +$	$0 - 0\ 0\ 0$
$0 - + - -$	$0 - + + +$	$0 - + + 0$	$0 - + + -$
$0\ 0 - + +$	$0\ 0 - 0\ 0$	$0\ 0 - 0 +$	$0\ 0 - 0 -$
$0\ 0 + 0\ 0$	$0\ 0 + - -$	$0\ 0\ 0\ 0\ 0$	$0\ 0 + - +$
$0 + - + 0$	$0 + - - +$	$0 + - - -$	$0 + - - 0$
$0 + 0 - -$	$0 + 0 + 0$	$0 + 0 + -$	$0 + 0 + +$
$+ - - 0 -$	$+ - - + 0$	$+ - - 0 -$	$+ - - + 0$
$+ - + + +$	$+ - 0 - +$	$+ - 0 + +$	$+ - 0 - +$
$+ 0\ 0 - 0$	$+ 0\ 0\ 0 +$	$+ 0\ 0 - 0$	$+ 0\ 0 + -$
$+ 0\ 0 + -$	$+ 0 + + -$	$+ 0 + - -$	$+ 0 + 0\ 0$
$+ + - - +$	$+ + - 0 -$	$+ + - + 0$	$+ + - 0 +$
$+ + + 0\ 0$	$+ + + - 0$	$+ + + 0 +$	$+ + + - -$

Table 6 gives the values of $Q(\Gamma^{(5)})$ for these designs, along with the corresponding averages of their four- and three-factor projections. It shows that designs 1 and 2 with low values of $Q(\Gamma^{(5)})$ have better projection properties than other designs. For information on designs for six factors in 18 runs, the reader is referred to Tsai et al. (2000).

Table 6. Properties for designs for five factors in 18 runs.

Design	$Q(\Gamma^{(5)})$	Average $Q(\Gamma^{(4)})$	Average $Q(\Gamma^{(3)})$
1	1.6018	0.9574	0.5288
2	1.6040	0.9683	0.5313
3	1.6141	0.9694	0.5329
28	1.6489	0.9734	0.5320
128	1.7051	0.9984	0.5382
129	1.7055	0.9853	0.5328
320	2.0079	1.0992	0.5574

3. Designs with Fewer Runs

Designs for four, five and six factors in 13 to 17 runs are generated and compared to the corresponding D-optimal main effects designs obtained from the SAS procedure OPTEX (SAS Institute, 1995, Pt.6). A brief summary of these results was given in Tsai et al. (2000). Here, we list some of the best designs for four, five and six factors in 13 to 17 runs.

Designs in 17 runs.

Lin (1993) found that the best two-level designs in $4n-1$ runs could be obtained by deleting any point from the best designs in $4n$ runs. Similarly we found that the best 17 run designs were obtained by deleting a point from the best 18-run designs. However, with the three-level factors it is important to delete an appropriate point. It turns out to be better to delete points with as many 0's as possible, rather than points with many ±1's.

The best design for three factors, with $Q(\Gamma^{(3)})$ equal to 0.5396, is obtained by deleting the point (–,0,0) or (+,0,0) from D1 in Table 1. The best design for four factors, with $Q(\Gamma^{(4)})$ equal to 0.9719, is obtained by deleting the point (–,0,0,0) or (+,0,0,0) from Design 1 in Table 3. The best design for five factors, with $Q(\Gamma^{(5)})$ equal to 1.6576, is obtained by deleting the point (0,0,–,0,0) from Design 2 in Table 5. For information on designs for six factors in 17 runs, the reader is referred to Tsai et al. (2000).

Designs in 16 runs.

Table 7 gives the plans for the best design for four, five and six factors in 16 runs under the Q criterion. Table 8 gives the values of the Q criterion and the D_s criterion

Table 7. Plans for best designs for four, five and six factors in 16 runs.

F=4	F=5	F=6
− − − −	− − − − −	− − − 0 0 +
− − 0 0	− − 0 + 0	− − + − − 0
− 0 − +	− 0 − 0 +	− 0 − 0 + −
− 0 + 0	− 0 + − 0	− 0 0 + − 0
− + 0 +	− + 0 0 −	− + 0 − 0 −
− + + −	− + + + +	− + + + + +
0 − 0 +	0 − 0 − +	0 − 0 − + +
0 − + −	0 − + 0 +	0 − + + 0 −
0 0 0 0	0 0 0 0 0	0 0 − − − +
0 0 + +	0 0 + + −	0 0 0 0 0 0
0 + − −	0 + − + 0	0 + − + − −
+ − − 0	+ − − + −	+ − − + + 0
+ − + +	+ − + 0 0	+ − 0 0 − −
+ 0 0 −	+ 0 0 + +	+ 0 + − + −
+ + − +	+ + − − +	+ + − − 0 0
+ + + 0	+ + + − −	+ + + 0 − +

Table 8. Properties of designs in 16 runs.

#Factor	Design	$Q(\Gamma^{(F)})$	$\log_{10}(D)$
F=4	1	1.0281	8.7815
	D	1.1419	8.8095
F=5	1	1.7391	10.6802
	D	1.9452	10.6802
F=6	1	2.2253	12.2463
	D	2.4274	12.5419

(on a log scale), excluding the intercept, of the main effects models for the best designs for 4 to 6 factors in 16 runs, along with those for the corresponding D-optimal designs. It shows that designs generated by the design procedure sacrifice a little D-efficiency for fitting the main effects model. However, they gain a lot over all possible models. The best designs that we generate appear clearly better than the D-optimal design with respect to the Q criterion.

Similar phenomena are observed for $n = 13, 14$ and 15 runs. Tables 9, 10 and 11 give plans for the best designs for 4 to 6 factors in 15, 14 and 13 runs respectively. Note also that for some D-optimal designs, the levels of some factors do not appear equally often, as nearly as possible. Unless one level of a factor is more important than the others, it is usually preferred that each level of a factor appears equally often, as nearly as possible.

Table 9. Plans for best designs for 4 to 6 factors in 15 runs.

F=4	F=5	F=6
− − − −	− − − + 0	− − − − − 0
− − 0 +	− − + − −	− − + 0 0 −
− 0 − +	− 0 − − +	− 0 − + + −
− 0 + −	− 0 + + +	− 0 0 − + +
− + 0 0	− + 0 0 0	− + 0 0 − +
0 − + 0	0 − 0 0 +	0 − − + 0 +
0 0 0 −	0 0 − 0 −	0 0 0 0 0 0
0 0 0 0	0 0 0 + 0	0 0 + − − −
0 + − −	0 + + − +	0 + − 0 + 0
0 + + +	0 + + + −	0 + + + + +
+ − − 0	+ − − − 0	+ − 0 − + −
+ − 0 +	+ − 0 + −	+ − + + − 0
+ 0 + 0	+ 0 + 0 0	+ 0 − 0 − +
+ + − +	+ + − 0 +	+ + 0 + 0 −
+ + + −	+ + 0 − −	+ + + − 0 0

Table 10. Plans for the best designs for 4 to 6 factors in 14 runs.

F=4	F=5	F=6
$- - - -$	$- - - - -$	$- - - - - 0$
$- - + 0$	$- - + + 0$	$- - + + 0 -$
$- 0 - +$	$- 0 - + +$	$- 0 - + + +$
$- + 0 -$	$- + 0 - +$	$- + 0 - + -$
$- + + +$	$- + + 0 -$	$- + + 0 - +$
$0 - 0 +$	$0 - + - +$	$0 - + - + +$
$0\ 0\ 0\ 0$	$0\ 0\ 0\ 0\ 0$	$0\ 0\ 0\ 0\ 0\ 0$
$0\ 0 + -$	$0\ 0\ 0\ 0\ 0$	$0\ 0\ 0\ 0\ 0\ 0$
$0 + - -$	$0 + - + -$	$0 + - + - -$
$+ - - 0$	$+ - - 0 +$	$+ - - 0 + -$
$+ - + +$	$+ - 0 + -$	$+ - 0 + - +$
$+ 0\ 0 -$	$+ 0 + - -$	$+ 0 + - - -$
$+ + - +$	$+ + - - 0$	$+ + - - 0 +$
$+ + + 0$	$+ + + + +$	$+ + + + + 0$

Table 11. Plans for the best designs for 4 to 6 factors in 13 runs.

F=4	F=5	F=6
$- - - -$	$- - 0\ 0 +$	$- - - 0 + 0$
$- - + 0$	$- - + - 0$	$- - + - - 0$
$- 0\ 0 +$	$- 0 - - -$	$- 0 + + 0 -$
$- + - +$	$- + - + 0$	$- + - - 0 -$
$- + + -$	$- + + 0 -$	$- + 0 + - +$
$0 - - +$	$0 - - + -$	$0 - 0 + - -$
$0\ 0\ 0\ 0$	$0\ 0\ 0\ 0\ 0$	$0\ 0 - 0 - +$
$0\ 0 + -$	$0\ 0 + + +$	$0\ 0\ 0\ 0\ 0\ 0$
$0 + 0\ 0$	$0 + - - +$	$0 + + - + +$
$+ - 0 -$	$+ - 0 - -$	$+ - - - 0 +$
$+ 0 - 0$	$+ 0 - 0\ 0$	$+ 0\ 0 - + -$
$+ + - -$	$+ + 0 + -$	$+ + - + + 0$
$+ + + +$	$+ + + - +$	$+ + + 0 - -$

4. Conclusion

In this paper, we have listed some of the best designs for 4 to 6 factors in 13 to 18 runs. The best four-factor designs in 18 runs generated by our procedure not only are the most efficient designs for fitting the main effects models, but also for the second-order model and any sub-model of it. For designs for five and six factors in 18 runs, for most subsets of factors the best designs have better projections and can provide better parameter estimates over a range of possible models that contain some interactions as well as main effects.

For designs in smaller run sizes, the best designs generated by the design procedure sacrifice a little efficiency for fitting the main effects model. However, they gain a lot over all possible models. Therefore, the best designs we have found deserve strong consideration in many situations.

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References

Cheng, S. W. and Wu, C. F. J. (2001). Factor screening and response surface exploration. *Statist. Sinica* **11**, 553-604.

Cheng, S. W. and Ye, K. Q. (2004). Geometric isomorphism and minimum aberration for factorial designs with quantitative factors. *Ann. Statist.* **32**, 2168-2185.

Hamada, M. and Wu, C. F. J. (1992). Analysis of designed experiments with complex aliasing. *J. Quality Technology* **24**, 130-137.

Lin, D. K. J. (1993). Another look at first-order saturated design: the p efficient designs. *Technometrics* **35**, 284-292.

SAS Institute (1995). *SAS/QC Software, Version* 6. Cary: SAS Institute Inc.

Tsai, P. W., Gilmour, S. G. and Mead, R. (2000). Projective three-level main effects designs robust to model uncertainty. *Biometrika* **87**, 467-475.

Wang, J. C. and Wu, C. F. J. (1995). A hidden projection property of Plackett-Burman and related designs. *Statist. Sinica* **5**, 235-250.

Wu, C. F. J. and Hamada, M. (2000). *Experiments.* Wiley, New York.

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