When is Acceleration Unnecessary in a Degradation Test: Supplementary Materials

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S.1 Introduction

This supplement provides proofs that are not included in the main paper. In Section S.9, we investigate the impacts of the failure threshold $\mathcal{D}_f$, the number of measurements m, the test duration t_M, and the quantile q on the necessity of acceleration.

S.2 Information matrix of $\boldsymbol{\theta}$ in a Wiener process

In a Wiener process, the second partial derivatives of $\ell(\boldsymbol{\theta})$ with respect to $\boldsymbol{\theta}$ are

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1^2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}\left[-\frac{2b^2\mu_i^{-2b}}{a^2\Delta t}(\Delta x_{ijl}-\mu_i\Delta t)^2 - \frac{(4b-1)\mu_i^{-2b+1}}{a^2}(\Delta x_{ijl}-\mu_i\Delta t) - \frac{\mu_i^{-2b+2}\Delta t}{a^2}\right],$$

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1\partial\delta_2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} s_i\left[-\frac{2b^2\mu_i^{-2b}}{a^2\Delta t}(\Delta x_{ijl}-\mu_i\Delta t)^2 - \frac{(4b-1)\mu_i^{-2b+1}}{a^2}(\Delta x_{ijl}-\mu_i\Delta t) - \frac{\mu_i^{-2b+2}\Delta t}{a^2}\right],$$

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1\partial a} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}\left[\frac{-2b\mu_i^{-2b}}{a^3\Delta t}(\Delta x_{ijl}-\mu_i\Delta t)^2 - \frac{2\mu_i^{-2b+1}}{a^3}(\Delta x_{ijl}-\mu_i\Delta t)\right],$$

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_2^2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} s_i^2\left[-\frac{2b^2\mu_i^{-2b}}{a^2\Delta t}(\Delta x_{ijl}-\mu_i\Delta t)^2 - \frac{(4b-1)\mu_i^{-2b+1}}{a^2}(\Delta x_{ijl}-\mu_i\Delta t) - \frac{\mu_i^{-2b+2}\Delta t}{a^2}\right],$$

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_2\partial a} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} s_i\left[\frac{-2b\mu_i^{-2b}}{a^3\Delta t}(\Delta x_{ijl}-\mu_i\Delta t)^2 - \frac{2\mu_i^{-2b+1}}{a^3}(\Delta x_{ijl}-\mu_i\Delta t)\right],$$

$$\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial a^2} = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}\left[\frac{1}{a^2} - \frac{3\mu_i^{-2b}}{a^4\Delta t}(\Delta x_{ijl}-\mu_i\Delta t)^2\right].$$

Under the Wiener process assumption, $E[\Delta X_{ijl}-\mu_i\Delta t]=0$, $E[(\Delta X_{ijl}-\mu_i\Delta t)^2]=\sigma_i^2\Delta t$, $\sum_{j=1}^{n_i}\sum_{l=1}^{m}\Delta t=\pi_i N t_M$, and $\sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}=mn$. The elements of the Fisher information matrix can be derived as

$$E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1^2}\right] = 2mnb^2 + \frac{nt_M}{a^2}\sum_{i=1}^{r}\pi_i\mu_i^{-2b+2},$$

$$E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1\partial\delta_2}\right] = 2mnb^2\sum_{i=1}^{r}s_i\pi_i + \frac{nt_M}{a^2}\sum_{i=1}^{r}s_i\pi_i\mu_i^{-2b+2},$$

$$E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1\partial a}\right] = \frac{2mnb}{a},$$

$$E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_2^2}\right] = 2mnb^2\sum_{i=1}^{r}s_i^2\pi_i + \frac{nt_M}{a^2}\sum_{i=1}^{r}s_i^2\pi_i\mu_i^{-2b+2},$$

$$E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_2\partial a}\right] = \frac{2mnb}{a}\sum_{i=1}^{r}s_i\pi_i,$$

$$E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial a^2}\right] = \frac{2mn}{a^2}.$$

S.3 Proof of $\mathbf{AVar_a}(\widehat{t_q}) > \mathbf{AVar_n}(\widehat{t_q})$ when $b = 1$ in a Wiener process

When the acceleration relation index $b = 1$, the Fisher information matrix $\mathcal{I}_W(\boldsymbol{\theta})$ is

$$\mathcal{I}_W(\boldsymbol{\theta}) = \frac{nt_M}{a^2}\begin{bmatrix} 1+p_2 & (1+p_2)\Sigma_1 & p_1 \\ & (1+p_2)\Sigma_2 & p_1\Sigma_1 \\ \text{symmetric} & & \frac{2m}{t_M} \end{bmatrix}, \tag{S.1}$$

where

$$\Sigma_1 = \sum_{i=1}^{r} s_i\pi_i,\ \Sigma_2 = \sum_{i=1}^{r} s_i^2\pi_i,\ p_1 = \frac{2mab}{t_M},\ p_2 = \frac{2ma^2b^2}{t_M}.$$

Substituting (S.1) into $\mathrm{AVar_a}(\widehat{t_q}) = \boldsymbol{h}'[\mathcal{I}(\boldsymbol{\theta})]^{-1}\boldsymbol{h}$ yields

$$\mathrm{Avar_a}(\widehat{t_q}) = \frac{a^2}{Nt_M}\frac{\frac{2m}{t_M}\left[\frac{\Sigma_2}{\Sigma_2-\Sigma_1^2}+p_2\right]h_1^2 - 2h_1h_3p_1 + h_3^2(1+p_2)}{\frac{2m}{t_M}(1+p_2)-p_1^2}. \tag{S.2}$$

On the other hand, the asymptotic variance of $\widehat{t_q}$ in the corresponding nonaccelerated test is

$$\mathrm{Avar_n}(\widehat{t_q}) = \frac{a^2}{Nt_M}\frac{\frac{2m}{t_M}(1+p_2)h_1^2 - 2h_1h_3p_1 + h_3^2(1+p_2)}{\frac{2m}{t_M}(1+p_2)-p_1^2}.$$

Note that $\Sigma_2 > 0$, $\Sigma_1^2 > 0$ for $0 \le s_i \le 1$ and $0 \le \pi_i \le 1$. Then,

$$\frac{\Sigma_2}{\Sigma_2 - \Sigma_1^2} > 1.$$

Therefore, when $b = 1$, $\mathrm{Avar_a}(\widehat{t_q}) > \mathrm{Avar_n}(\widehat{t_q})$ for all $\boldsymbol{\theta}$ in a Wiener process.

S.4 Proof of $\delta_2^* \equiv 1.28$ when $b = 0$ in a Wiener process

When the acceleration relation index $b = 0$, the Fisher information matrix $\mathcal{I}_W(\boldsymbol{\theta})$ is

$$\mathcal{I}_W(\boldsymbol{\theta}) = \frac{nt_M}{a^2}\begin{bmatrix} I_{11} & I_{12} & 0 \\ I_{12} & I_{22} & 0 \\ 0 & 0 & \frac{2m}{t_M} \end{bmatrix},$$

where

$$I_{11} = \sum_{i=1}^{r} \pi_i \mu_i^2, \;\; I_{12} = \sum_{i=1}^{r} s_i \pi_i \mu_i^2, \;\; I_{22} = \sum_{i=1}^{r} s_i^2 \pi_i \mu_i^2.$$

The inverse of $\mathcal{I}_W(\boldsymbol{\theta})$ is

$$[\mathcal{I}_W(\boldsymbol{\theta})]^{-1} = \frac{a^2}{nt_M}\begin{bmatrix} \frac{I_{22}}{I_{11}I_{22}-I_{12}^2} & \frac{-I_{12}}{I_{11}I_{22}-I_{12}^2} & 0 \\ & \frac{I_{11}}{I_{11}I_{22}-I_{12}^2} & 0 \\ \text{symmetric} & & \frac{t_M}{2m} \end{bmatrix}. \tag{S.3}$$

Substituting (S.3) into $\mathrm{AVar_a}(\widehat{t_q}) = \boldsymbol{h}'[\mathcal{I}(\boldsymbol{\theta})]^{-1}\boldsymbol{h}$ yields

$$\begin{aligned} \mathrm{Avar_a}(\widehat{t_q}) &= \frac{a^2}{Nt_M}\left(h_1^2 \frac{I_{22}}{I_{11}I_{22} - I_{12}^2} + h_3^2 \frac{t_M}{2m}\right) \\ &= \frac{a^2}{Nt_M}\left[h_1^2 \exp(-2\delta_1) \frac{\sum_{i=1}^{r} \pi_i \exp(2\delta_2 s_i) s_i^2}{\sum_{i<l}^{r} \pi_i \pi_l \exp(2\delta_2 s_i + 2\delta_2 s_l)(s_l - s_i)^2} + h_3^2 \frac{t_m}{2m}\right] \\ &= \frac{a^2}{Nt_M}\left[h_1^2 \exp(-2\delta_1) \varSigma + h_3^2 \frac{t_m}{2m}\right]. \end{aligned} \tag{S.4}$$

where

$$\varSigma = \frac{\sum_{i=1}^{r} \pi_i \exp(2\delta_2 s_i) s_i^2}{\sum_{i<l}^{r} \pi_i \pi_l \exp(2\delta_2 s_i + 2\delta_2 s_l)(s_l - s_i)^2}.$$

On the other hand, the asymptotic variance of $\widehat{t_q}$ in the corresponding nonaccelerated test is

$$\mathrm{Avar_n}(\widehat{t_q}) = \frac{a^2}{nt_M}\left[h_1^2 \exp(-\delta_1) + h_3^2 \frac{t_M}{2m}\right]. \tag{S.5}$$

The difference between (S.4) and (S.5) is $\varSigma$. Since $\varSigma$ is not related to δ_1 and a, the value of δ_2^* is thereby not related to δ_1 and a. Numerical results show that δ_2^* is around 1.28 in this case.

S.5 Information matrix of $\boldsymbol{\theta}$ in a gamma process

Under the gamma process assumption, the log-likelihood based on $\mathbf{D}$ (up to a constant) can be written as:

$$\ell(\boldsymbol{\theta}) = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}\left[-\ln\{\Gamma(k_i\Delta t)\} - k_i\Delta t \ln\theta_i + k_i\Delta t\ln(\Delta x_{ijl}) - \frac{\Delta x_{ijl}}{\theta_i}\right].$$

Note that $E[\Delta x_{ijl} - k_i\theta_i\Delta t] = 0$, and $E\{\ln(\Delta x_{ijl}) - [\psi(k_i\Delta t) + \ln\theta_i]\} = 0$ with the digamma function $\psi(\cdot)$. Elements of the Fisher information matrix of $\boldsymbol{\theta}$ can be derived as

$$
\begin{aligned}
E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1^2}\right] &= 4(1-b)^2\frac{nt_M^2}{a^4m}\sum_{i=1}^{n}\pi_i\psi_1\mu_i^{4(1-b)} + (3-2b)(2b-1)\frac{nt_M}{a^2}\sum_{i=1}^{r}\pi_i\mu_i^{2(1-b)},\\
E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1\partial\delta_2}\right] &= 4(1-b)^2\frac{nt_M^2}{a^4m}\sum_{i=1}^{n}s_i\pi_i\psi_1\mu_i^{4(1-b)} + (3-2b)(2b-1)\frac{nt_M}{a^2}\sum_{i=1}^{r}s_i\pi_i\mu_i^{2(1-b)},\\
E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_1\partial a}\right] &= -4(1-b)\frac{nt_M^2}{a^5m}\sum_{i=1}^{n}\pi_i\psi_1\mu_i^{4(1-b)} + 4(1-b)\frac{nt_M}{a^3}\sum_{i=1}^{r}\pi_i\mu_i^{2(1-b)},\\
E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_2^2}\right] &= 4(1-b)^2\frac{nt_M^2}{a^4m}\sum_{i=1}^{n}s_i^2\pi_i\psi_1\mu_i^{4(1-b)} + (3-2b)(2b-1)\frac{nt_M}{a^2}\sum_{i=1}^{r}s_i^2\pi_i\mu_i^{2(1-b)},\\
E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial\delta_2\partial a}\right] &= -4(1-b)\frac{nt_M^2}{a^5m}\sum_{i=1}^{n}s_i\pi_i\psi_1\mu_i^{4(1-b)} + 4(1-b)\frac{nt_M}{a^3}\sum_{i=1}^{r}s_i\pi_i\mu_i^{2(1-b)},\\
E\left[-\frac{\partial^2\ell(\boldsymbol{\theta})}{\partial a^2}\right] &= \frac{4nt_M^2}{a^6m}\sum_{i=1}^{n}\pi_i\psi_1\mu_i^{4(1-b)} - \frac{4nt_M}{a^4}\sum_{i=1}^{r}\pi_i\mu_i^{2(1-b)},
\end{aligned}
$$

where $\psi_1 = \psi_1(\mu_i^{2-2b}\Delta t/a^2)$ with the trigamma function $\psi_1(\cdot)$.

S.6 Proof of $\mathbf{AVar_a}(\widehat{t_q}) > \mathbf{AVar_n}(\widehat{t_q})$ when $b = 1$ in a gamma process

When the acceleration relation index $b = 1$, the Fisher information matrix $\mathcal{I}_{Ga}(\boldsymbol{\theta})$ is

$$
\mathcal{I}_{Ga}(\boldsymbol{\theta}) = \frac{4nt_M^2}{a^2m}\begin{bmatrix} p & p\Sigma_1 & 0 \\ p\Sigma_1 & p\Sigma_2 & 0 \\ 0 & 0 & \frac{\psi_1}{a^4} - \frac{m}{a^2t_M} \end{bmatrix}, \tag{S.6}
$$

where

$$
\Sigma_1 = \sum_{i=1}^{r}s_i\pi_i,\ \Sigma_2 = \sum_{i=1}^{r}s_i^2\pi_i,\ p = \frac{m}{4t_M},\ \psi_1 = \psi_1(\Delta t/a^2).
$$

Substituting (S.6) into $\text{AVar}_{\text{a}}(\widehat{t_q}) = \boldsymbol{h}'[\mathcal{I}(\boldsymbol{\theta})]^{-1}\boldsymbol{h}$ yields

$$\text{Avar}_{\text{a}}(\widehat{t_q}) = \frac{a^2 m}{4nt_M^2}\left[\frac{4t_M}{m}h_1^2\frac{\Sigma_2}{\Sigma_2 - \Sigma_1^2} + \frac{h_3^2}{\frac{\psi_1}{a^4} - \frac{m}{a^2 t_M}}\right]. \tag{S.7}$$

On the other hand, the asymptotic variance of $\widehat{t_q}$ in the corresponding nonaccelerated test is

$$\text{Avar}_{\text{n}}(\widehat{t_q}) = \frac{a^2 m}{4nt_M^2}\left[\frac{4t_M}{m}h_1^2 + \frac{h_3^2}{\frac{\psi_1}{a^4} - \frac{m}{a^2 t_M}}\right].$$

Note that $\Sigma_2 > 0$, $\Sigma_1^2 > 0$ for $0 \le s_i \le 1$ and $0 \le \pi_i \le 1$. Then,

$$\frac{\Sigma_2}{\Sigma_2 - \Sigma_1^2} > 1.$$

Therefore, when $b = 1$, $\text{Avar}_{\text{a}}(\widehat{t_q}) > \text{Avar}_{\text{n}}(\widehat{t_q})$ for all $\boldsymbol{\theta}$ under the gamma process assumption.

S.7 Information matrix of $\boldsymbol{\theta}$ in an IG process

Under the IG process assumption, the log-likelihood based on $\mathbf{D}$ (up to a constant) is given by

$$\ell(\boldsymbol{\theta}) = \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{k=1}^{m}\left[\frac{3-2b}{2}\ln\mu_i - \ln a - \frac{\mu_i^{1-2b}(\Delta x_{ijl} - \mu_i\Delta t)^2}{2a^2\Delta x_{ijl}}\right].$$

It is readily shown that $E[\Delta x_{ijl} - \mu_i\Delta t] = 0$, $E[\Delta x_{ijl} - \mu_i\Delta t]^2 = \sigma_i^2 t = a^2\mu_i^{2b}\Delta t$, and $E\left[\frac{1}{\Delta x_{ijl}}\right] = \frac{1}{\alpha_i\Delta t} + \frac{1}{\beta_i\Delta t^2} = \frac{\mu_i^{-1}}{\Delta t} + \frac{a^2}{(\Delta t)^2}\mu_i^{2b-3}$. Therefore, the elements of the Fisher information matrix of $\boldsymbol{\theta}$ in an IG process are

$$
\begin{aligned}
E\left[-\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_1^2}\right] &= \frac{nt_M}{a^2}\sum_{i=1}^{r}\pi_i\mu_i^{2-2b} + \frac{mn}{2}(3-4b+4b^2),\\
E\left[-\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_1\partial\delta_2}\right] &= \frac{nt_M}{a^2}\sum_{i=1}^{r}s_i\pi_i\mu_i^{2-2b} + \frac{mn}{2}(3-4b+4b^2)\sum_{i=1}^{r}s_i\pi_i,\\
E\left[-\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_1\partial a}\right] &= \frac{mn}{a}(2b-1),\\
E\left[-\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_2^2}\right] &= \frac{nt_M}{a^2}\sum_{i=1}^{r}s_i^2\pi_i\mu_i^{2-2b} + \frac{mn}{2}(3-4b+4b^2)\sum_{i=1}^{r}s_i^2\pi_i,\\
E\left[-\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_2\partial a}\right] &= \frac{mn}{a}(2b-1)\sum_{i=1}^{r}s_i\pi_i,\\
E\left[-\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial a^2}\right] &= \frac{2mn}{a^2}.
\end{aligned}
$$

S.8 Information matrix of $\boldsymbol{\theta}$ in an ED model

In an ED model, the second derivatives of $\ell(\boldsymbol{\theta})$ with respect of $\boldsymbol{\theta}$ are given by

$$\begin{aligned}
\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_1^2} &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}\bigg\{ \frac{(d-2b)^2\mu_i^{d-2b}}{a^2}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \\
&\qquad + \frac{(d-4b+1)\mu_i^{-2b+1}}{a^2}[\Delta x_{ijl} - \kappa'(\varpi_i)\Delta t] - \frac{\mu_i^{-2b+2}\Delta t}{a^2}\bigg\}, \\
\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_1 \partial \delta_2} &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} s_i \bigg\{ \frac{(d-2b)^2\mu_i^{d-2b}}{a^2}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \\
&\qquad + \frac{(d-4b+1)\mu_i^{-2b+1}}{a^2}[\Delta x_{ijl} - \kappa'(\varpi_i)\Delta t] - \frac{\mu_i^{-2b+2}\Delta t}{a^2}\bigg\}, \\
\frac{\partial \ell(\boldsymbol{\theta})}{\partial \delta_1 \partial a} &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m}\bigg\{ -\frac{2(d-2b)\mu_i^{d-2b}}{a^3}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \\
&\qquad - \frac{2\mu_i^{-2b+1}}{a^3}[\Delta x_{ijl} - \kappa'(\varpi_i)\Delta t]\bigg\}, \\
\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial \delta_2^2} &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} s_i^2 \bigg\{ \frac{(d-2b)^2\mu_i^{d-2b}}{a^2}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \\
&\qquad + \frac{(d-4b+1)\mu_i^{-2b+1}}{a^2}[\Delta x_{ijl} - \kappa'(\varpi_i)\Delta t] - \frac{\mu_i^{-2b+2}\Delta t}{a^2}\bigg\}, \\
\frac{\partial \ell(\boldsymbol{\theta})}{\partial \delta_2 \partial a} &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} s_i \bigg\{ -\frac{2(d-2b)\mu_i^{d-2b}}{a^3}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \\
&\qquad - \frac{2\mu_i^{-2b+1}}{a^3}[\Delta x_{ijl} - \kappa'(\varpi_i)\Delta t]\bigg\}, \\
\frac{\partial^2 \ell(\boldsymbol{\theta})}{\partial a^2} &= \sum_{i=1}^{r}\sum_{j=1}^{n_i}\sum_{l=1}^{m} \left\{ \frac{1}{a^2} + \frac{6\mu_i^{d-2b}}{a^4}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \right\}.
\end{aligned}$$

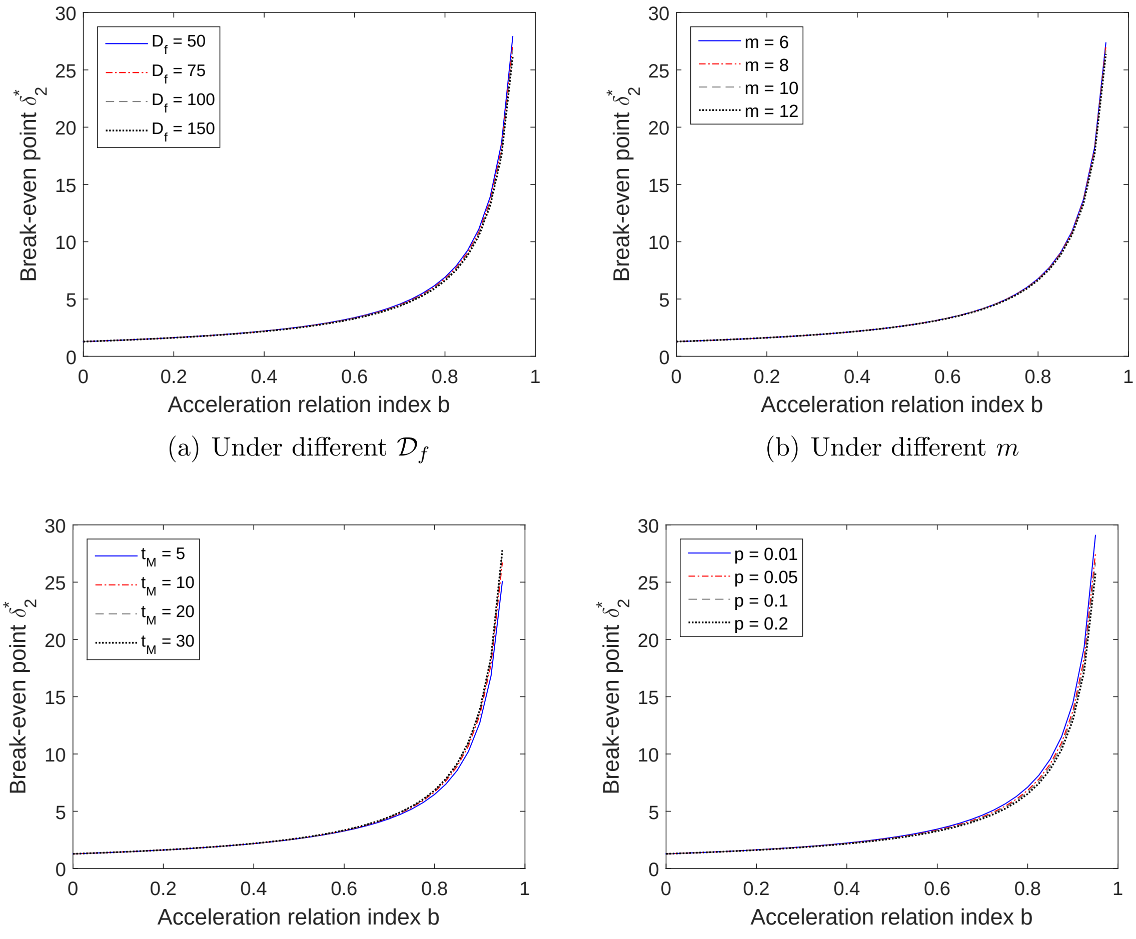
Note that $E[\Delta x_{ijl} - \kappa'(\varpi_i)\Delta t] = 0$. Let

$$C_i = E\left\{ \sum_{j=1}^{n_i}\sum_{l=1}^{m} \mu_i^{d-2b}[\varpi_i \Delta x_{ijl} - \kappa(\varpi_i)\Delta t - C_{ijl}] \right\}.$$

Elements of the Fisher information matrix in an ED model can be obtained.

S.9 Impacts of $\mathcal{D}_f$, m, t_M and q on the necessity of acceleration

In this section, we investigate the effects of the failure threshold $\mathcal{D}_f$, number of measurements m, test duration t_M and the quantile q on the necessity of acceleration. When the acceleration relation index $b \geq 1$, the necessity of acceleration is not related to these parameters. That is because the signal-to-noise ratio always decreases with the stress levels even under different settings of these parameters. Therefore, the conclusion that acceleration is unnecessary when $b \geq 1$ is valid for all settings of $\mathcal{D}_f$, m, t_M and q. When the acceleration relation index $b < 1$, we calculate the numerical values of the break-even point δ_2^* under different settings of these parameters in a two-stress level ADT. Without loss of generality, we set $\mathcal{D}_f = 100$, $m = 10$, $t_M = 10$ and $q = 0.1$ as the baseline. Usually, we are concerned about a small quantile q, as mature products are expected to have low failure rates within the mission time. Based on this consideration, we set $q = 0.01$, 0.05, 0.1 and 0.2. Similarly, parameter settings of $\mathcal{D}_f$, m and t_M are also chosen based on the values commonly used in practical applications. Figure S.1 plots the patterns of δ_2^* under different settings of $\mathcal{D}_f$, m, t_M and q in a Wiener process when $b \in [0, 1)$. As can be seen, the value of δ_2^* is not sensitive to these parameters. Numerical results of the gamma and IG processes are similar to those of the Wiener process and thus are omitted here. Therefore, the parameters $\mathcal{D}_f$, m, t_M and q have negligible effects on the necessity of acceleration.



(a) Under different $\mathcal{D}_f$

(b) Under different m

(c) Under different t_M

(d) Under different q

Figure S.1: Values of the break-even point δ_2^* under different settings of $\mathcal{D}_f$, m, t_M and q when the acceleration relation index $b \in [0, 1)$ in a Wiener process ($a = 0.5$, $\delta_1 = 1$).