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A GENERAL MINIMUM LOWER-ORDER CONFOUNDING CRITERION FOR TWO-LEVEL REGULAR DESIGNS

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Abstract: Based on the effect hierarchy principle in experimental design, an aliased effect-number pattern (AENP, or AP for short) is proposed to judge two-level regular designs; it contains the basic information of all effects aliased with other effects at varying severity degrees in a design. Based on the AENP, a general minimum lower-order confounding (GMLOC, or GMC for short) criterion is proposed, and several results follow. First, the word-length pattern, as the core of the minimum aberration (MA) criterion, is a function of the AENP. The same also holds for the clear effects (CE) criterion. Furthermore, the estimation capacity (EC) of a design can be also calculated as a function of the new pattern, and links between the MA and CE criteria are discovered. In addition, a concept of estimation ability is introduced, and it is concluded that a GMC design is the one with the best estimation ability. Finally, more applications of the new pattern are given. All GMC designs of 16 and 32 runs, a number of GMC designs of 64 runs, and some comparisons with the optimal designs under MA and CE criteria are tabulated.

Key words and phrases: Clear effects criterion, effect hierarchy principle, estimation ability, estimation capacity, minimum aberration.

1. Introduction

One of the main tasks in experimental design is to find good designs and to analyze experimental data effectively, so that more effects and more possible models related to the effects in experiments can be estimated. Regular designs have been the most commonly considered designs in practice, due to their simple confounding structure.

The effect hierarchy principle states that a lower-order effect is likely more important than a higher-order one, and effects of the same order are equally important. Therefore, to estimate more important parameters and models, a good design should minimize the confounding between the lower-order effects.

In this paper, we only discuss the case of two-level regular designs. A regular 2^{n-m} design is determined by m independent defining words and all possible products of the m words constitute a subgroup, denoted by $G = \{I, w_1, \ldots,$

$w_{2^m-1}\}$. Starting from the subgroup, there are several optimality criteria for choosing good designs. We focus on four of them.

The first is the maximum resolution (MR) criterion proposed by Box and Hunter (1961). This criterion chooses the good designs with MR, but does not distinguish among them.

Going further, Fries and Hunter (1980) proposed the minimum aberration (MA) criterion. It is based on the word-length pattern (WLP)

$$W = (A_1, A_2, A_3, A_4, \ldots, A_n), \tag{1.1}$$

where A_i denotes the number of words with length i in G. A design sequentially minimizing the A_i's in the WLP is called an MA design. In the past two and half decades, much attention has been paid to the theory and construction of MA designs. Many related papers have been published, including Franklin (1984), Chen and Wu (1991), Chen, Sun and Wu (1993), Chen and Hedayat (1996), Tang and Wu (1996), Suen, Chen and Wu (1997), Zhang and Park (2000), Zhang and Shao (2001), Butler (2003), Ai and Zhang (2004a), Zhu and Zeng (2005), Cheng and Tang (2005) and Xu (2006). A nice summary of MA designs is given in Mukerjee and Wu (2006).

A third option is the clear effects (CE) criterion. Wu and Chen (1992) first introduced the notion of clear effects and noticed that the MA criterion sometimes does not result in satisfactory designs. The CE criterion selects designs that sequentially maximize the numbers of clear main effects and clear two-factor interactions (2fi's). Recent results in this direction include Chen and Hedayat (1998), Tang, Ma, Ingram and Wang (2002), Wu and Wu (2002), Ai and Zhang (2004b), Yang, Liu and Zhang (2005), Yang, Li, Liu and Zhang (2006) and Chen, Li, Liu and Zhang (2006). However, the CE criterion is only applicable to the designs having clear effects and cannot be used to compare designs having the same numbers of clear main effects and 2fi's. Many examples of optimal CE designs that differ from MA designs have been found by investigators (see Wu and Hamada (2000), Li, Chen, Liu and Zhang (2006) and Tables 3 and 4 in the Appendix).

A fourth criterion is estimation capacity (EC), first introduced by Sun (1993). Its idea is to estimate as many as possible models involving all the main effects and some 2fi's. Cheng and Mukerjee (1998), Cheng, Steinberg and Sun (1999) and Ai and Zhang (2004c) have studied it in detail, and obtained some "good" designs with maximum estimation capacity (MEC). Note that for MEC, the estimability of effects requires all the 2fi's not in the model to be absent (Mukerjee and Wu (2006)).

Facing many criteria, one can ask several questions. What relationships are there among the criteria? Why do the criteria originating from the same ideas, such as the MA and CE criteria, often give different optimal designs? What is the basic information contained in the subgroup G? Is there a criterion that can more reasonably reflect the effect hierarchy principle? In this paper we try to answer these questions.

In Section 2 we introduce a new aliasing pattern, denoted as AENP, and based on it propose a general minimum lower-order confounding (GMC) criterion for rank-ordering regular 2^{n-m} designs. The relationships of the new criterion with the MA, CE and EC criteria are studied in Sections 3, 4 and 5, respectively. The links between the MA and CE criteria are addressed in Section 4. A novel criterion, the maximum estimation ability criterion, is proposed in Section 6. In Section 7 we simplify the AENP and provide more applications via examples. All the GMC designs of 16 and 32 runs, a number of GMC designs of 64 runs, and some comparisons with the MA and CE criteria are tabulated in the Appendix. Some explanations are given in Section 8.

2. A New Aliasing Pattern and a General Minimum Lower-Order Confounding Criterion

In order to give a reasonable aliasing pattern, we need to further explore the basic information hidden in the subgroup G. For a given ordered pair (i, j), to describe how the ith-order effects are to be aliased with the jth-order effects, we need to consider two basic elements. First, for a given ith-order effect, assess how severely it is aliased with the jth-order effects and measure the aliased severity degree. If the ith-order effect is aliased with k jth-order effects simultaneously, we can say that the degree of the ith-order effect being aliased with the jth-order effects is k. The second consideration is how many ith-order effects are aliased with the jth-order effects at a given degree k. We use the notation ${}_{i}^{\#}C_{j}^{(k)}$ to denote the number of ith-order effects aliased with jth-order effects at degree k. Thus, for a design, we have a set

$$\{{}_{i}^{\#}C_{j}^{(k)}, i, j = 0, 1, \ldots, n, k = 0, 1, \ldots, K_j\}, \tag{2.1}$$

where $K_j = \binom{n}{j}$. The set reflects the overall confounding between effects in a design. Note that the numbers in (2.1) are not symmetric with respect to i and j (see Example 2 for an illustration).

The numbers in (2.1) are not equally important and should be usefully arranged. Clearly, for an ith-order effect, the lesser the degree at which it is aliased with other effects, the more easily it can be estimated. In particular if it is aliased at degree 0 with lower-order effects, and higher-order effects are negligible, then it can be estimated without confounding. In addition, since the total number of ith-order effects in a 2^{n-m} design is $\binom{n}{i}$, the larger the number ${}_{i}^{\#}C_{j}^{(0)}$, the less severely ith-order effects are confounded by jth-order effects. Subsequently, under the condition of maximizing the number ${}_{i}^{\#}C_{j}^{(0)}$, the larger the number ${}_{i}^{\#}C_{j}^{(1)}$, the less severely ith-order effects are confounded with jth-order effects, and so on. Consider $\{{}^{\#}_{i}C_j^{(k)},\ k = 0, 1, \ldots, K_j\}$. Since the larger the degree k, the more severely the effect is aliased, we should rank the numbers of aliased ith-order effects with jth-order effects from degree 0 to the most severe degree in the order

$$ {}^{\#}_{i}C_j = ({}^{\#}_{i}C_j^{(0)}, \ldots, {}^{\#}_{i}C_j^{(K_j)}), \tag{2.2} $$

which simply shows a distribution of the numbers of ith-order effects aliased with jth-order effects on the degrees $k = 0, 1, \ldots, K_j$. Note that the 0th-order effect is the grand mean. To save space, for a vector ${}^{\#}_{i}C_j$, we use 0^s to denote s successive zero components in it and if it has a tail with successive zero components we cut the tail part hereafter.

Consider the ranking of the different vectors ${}^{\#}_{i}C_j$'s. First we ignore ${}^{\#}_{0}C_0$, ${}^{\#}_{0}C_1$ and ${}^{\#}_{1}C_0$ since ${}^{\#}_{0}C_0 = (1)$, ${}^{\#}_{0}C_1 = (1)$ and ${}^{\#}_{1}C_0 = (n)$ for the 2^{n-m} designs. According to the effect hierarchy principle, we should rank ${}^{\#}_{1}C_1$ first, and then consider the vectors related to 2fi's. For every $i \geq 2$, consider the two vectors ${}^{\#}_{0}C_i = (0^{A_i}, 1)$ and ${}^{\#}_{i}C_0 = ({}^{\#}_{i}C_0^{(0)}, {}^{\#}_{i}C_0^{(1)})$. Obviously ${}^{\#}_{0}C_i$ should be placed before ${}^{\#}_{i}C_0$ because the 0th-order effect is more important. Since the latter can be determined by the former for every i, we can ignore all ${}^{\#}_{i}C_0$'s. Next, if the 2fi's are not negligible, then we should rank the vectors ${}^{\#}_{0}C_2$, ${}^{\#}_{1}C_2$, ${}^{\#}_{2}C_1$ and ${}^{\#}_{2}C_2$ in order as $({}^{\#}_{0}C_2,\ {}^{\#}_{1}C_2,\ {}^{\#}_{2}C_1,\ {}^{\#}_{2}C_2)$. The reason for placing ${}^{\#}_{0}C_2$ first is related to whether the grand mean effect can be estimated under the assumption that 2fi's cannot be neglected; putting ${}^{\#}_{1}C_2$ before ${}^{\#}_{2}C_1$ is due to the fact that main effects are more important than 2fi's; ${}^{\#}_{2}C_2$ should be placed last. If the third-order effects are not negligible, following the arguments above, we should rank the vectors ${}^{\#}_{0}C_3$, ${}^{\#}_{1}C_3$, ${}^{\#}_{2}C_3$, ${}^{\#}_{3}C_1$, ${}^{\#}_{3}C_2$ and ${}^{\#}_{3}C_3$ in order as $({}^{\#}_{0}C_3,\ {}^{\#}_{1}C_3,\ {}^{\#}_{2}C_3,\ {}^{\#}_{3}C_1,\ {}^{\#}_{3}C_2,\ {}^{\#}_{3}C_3)$, and so on. The general rule can be described as follows: (i) if $\max(i, j) < \max(s, t)$ then ${}^{\#}_{i}C_j$ is placed ahead of ${}^{\#}_{s}C_t$; (ii) if $\max(i, j) = \max(s, t)$ and $i < s$ then ${}^{\#}_{i}C_j$ is placed ahead of ${}^{\#}_{s}C_t$; (iii) if $\max(i, j) = \max(s, t)$, $i = s$ and $j < t$, then ${}^{\#}_{i}C_j$ is placed ahead of ${}^{\#}_{s}C_t$. Therefore, according to the effect hierarchy principle we rank the numbers at (2.1) as

$$ \begin{aligned} {}^{\#}C = ({}^{\#}_{1}C_1,\ &{}^{\#}_{0}C_2,\ {}^{\#}_{1}C_2,\ {}^{\#}_{2}C_1,\ {}^{\#}_{2}C_2,\ {}^{\#}_{0}C_3,\ {}^{\#}_{1}C_3,\ {}^{\#}_{2}C_3,\ {}^{\#}_{3}C_1,\ {}^{\#}_{3}C_2,\ {}^{\#}_{3}C_3, \\ &{}^{\#}_{0}C_4,\ {}^{\#}_{1}C_4,\ {}^{\#}_{2}C_4,\ {}^{\#}_{3}C_4,\ {}^{\#}_{4}C_1,\ {}^{\#}_{4}C_2,\ {}^{\#}_{4}C_3,\ {}^{\#}_{4}C_4, \ldots). \end{aligned} \tag{2.3} $$

We call the ordering (2.3) an aliased effect-number pattern (AENP), or aliasing pattern (AP) for short. Such a pattern, as well as (2.1), contains the basic information of all effects aliased with other effects at varying degrees in a design.

A simple and quick algorithm for calculating the AENP of any design through its defining pencil matrix is available from the authors.

A main purpose of experimental design is to estimate as many factorial effects as possible, especially the lower-order effects, e.g., the main effects and 2fi's. So, a "good" design should minimize the confounding between the lower-order effects and hence should maximize the entries of ${}^{\#}C$ sequentially. We define the new criterion as follows.

Definition 1. Let ${}^{\#}C_l$ be the l-th component of ${}^{\#}C$, and ${}^{\#}C(d)$ and ${}^{\#}C(d')$ the AENPs of designs d and d', respectively. Suppose that ${}^{\#}C_l$ is the first component such that ${}^{\#}C_l(d)$ and ${}^{\#}C_l(d')$ are different. If ${}^{\#}C_l(d) > {}^{\#}C_l(d')$, then d is said to have less general lower-order confounding (GLOC) than d'. A design d is said to have general minimum lower-order confounding (GMLOC, or GMC for short) if no other design has less GLOC than d and such a design is called a GMC design.

The following theorem follows directly from the definition of GMC.

Theorem 1. *A GMC 2^{n-m} design must have maximum resolution among all 2^{n-m} designs.*

3. Relationship with Minimum Aberration Criterion

In order to study the relationship between the GMC and MA criterion, we need to understand the relationship between WLP and AENP as the cores of MA and GMC respectively.

Theorem 2. *For a 2^{n-m} design with $R \geq III$, its WLP in* (1.1) *is a function of $\{{}^{\#}_{i}C^{(k)}_{j},\ i,j = 0,\ldots,n, k = 1,\ldots,K_j\}$ in the following two forms:*

(1) ${}^{\#}_{i}C^{(0)}_{0} = \binom{n}{i} - A_i$ *or* ${}^{\#}_{i}C^{(1)}_{0} = A_i$;

(2) *For any i, A_i is a function of ${}_sC_t, s, t = 1,\ldots,n$, in* (3.2), *where ${}_sC_t$ is a function of $\{{}^{\#}_{s}C^{(k)}_{t}, k = 1,\ldots,K_t\}$ as in* (3.3), *and sequentially minimizing A_i's of W is equivalent to sequentially minimizing ${}_sC_t$'s of C in* (3.2).

Proof. By the definition of the AENP, part (1) of the theorem is trivial.

For a 2^{n-m} design with $R \geq III$, Zhang and Park (2000) defined ${}_iC_j$ as the number of alias relations between ith-order and jth-order effects in a design, and obtained, for $i \leq j$,

$$ {}_iC_j = \sum_{l=0}^{i} \binom{n-(j-i+2l)}{i-l}\binom{j-i+2l}{l} A_{j-i+2l}, \quad i,j = 1,\ldots,n, \tag{3.1} $$

where $\binom{x}{0} = 1$, $\binom{x}{y} = 0$ for $x < y$ or $x < 0$, and $A_i = 0$ for $i \leq 2$ or $i > n$.

Furthermore, they proposed using the sequence

$$ C = ({}_1C_1,\ {}_1C_2,\ {}_2C_2,\ {}_1C_3,\ {}_2C_3,\ {}_3C_3,\ {}_1C_4,\ {}_2C_4,\ {}_3C_4,\ {}_4C_4,\ \ldots) \tag{3.2} $$

to choose optimal designs. Based on (3.1), they showed that sequences (1.1) and (3.2) can be determined from each other, and that sequentially minimizing (3.2) is equivalent to sequentially minimizing (1.1).

By the definition of ${}_iC_j$, and comparing with the definition of alias sets for a regular design, it is easy to get the following relations for all i, j:

$$ {}_iC_j = \begin{cases} \displaystyle\sum_{k=1}^{K_i} \frac{k\,{}^{\#}_{i}C_i^{(k)}}{2}, & \text{if } i = j, \\ \displaystyle\sum_{k=1}^{K_j} k\,{}^{\#}_{i}C_j^{(k)}, & \text{if } i \neq j. \end{cases} \tag{3.3} $$

Thus (2) is proved.

From Theorem 2, we have the following corollary.

Corollary 1. *The designs with different WLPs must have different AENPs.*

The converse of the corollary does not hold, designs with different AENPs may have the same WLP. The following is an example of this.

Example 1. Consider the two 2^{12-7} designs:

$$d_1 : I = 126 = 137 = 238 = 12349 = 1235t_0 = 45t_1 = 12345t_2,$$
$$d_2 : I = 126 = 137 = 248 = 349 = 125t_0 = 135t_1 = 145t_2,$$

where t_0, t_1 and t_2 denote the factors 10, 11 and 12. The designs d_1 and d_2 have $W = (0, 0, 8, 15, 24, 32, 24, 15, 8, 0, 0, 1)$, but their AENPs are different. In particular, they first differ at ${}^{\#}_{2}C_2^{(1)}(d_1) = 60$ and ${}^{\#}_{2}C_2^{(1)}(d_2) = 54$.

Consequently, the AENP is a more refined pattern than the WLP for judging designs; the WLP is only related to the portion $\{{}^{\#}_{i}C_0^{(1)}, i = 1, \ldots\}$ of the AENP.

On the other hand, from part (2) of Theorem 2, we can see that the MA criterion only uses information from $\{{}^{\#}_{i}C_j^{(k)}, i, j = 0, 1, \ldots, n, k = 1, \ldots, K_j\}$ without $\{{}^{\#}_{i}C_j^{(0)}, i, j = 0, 1, \ldots, n, \}$. We note that although ${}^{\#}_{i}C_j^{(0)}$ can determine the sum $\sum_{k=1}^{K_j} {}^{\#}_{i}C_j^{(k)}$, it cannot determine the vector $({}^{\#}_{i}C_j^{(1)}, \ldots, {}^{\#}_{i}C_j^{(K_j)})$ and ${}_iC_j = \sum_{k=1}^{K_j} k\,{}^{\#}_{i}C_j^{(k)}$. Therefore, it is possible for two designs d and d' with ${}^{\#}_{i}C_j^{(0)}(d) > {}^{\#}_{i}C_j^{(0)}(d')$ to have $\sum_{k=1}^{K_j} {}^{\#}_{i}C_j^{(k)}(d) < \sum_{k=1}^{K_j} {}^{\#}_{i}C_j^{(k)}(d')$, and at the same time to have ${}_iC_j(d) = \sum_{k=1}^{K_j} k\,{}^{\#}_{i}C_j^{(k)}(d) > {}_iC_j(d') = \sum_{k=1}^{K_j} k\,{}^{\#}_{i}C_j^{(k)}(d')$.

Consider the two designs d_6 and d_7 in Example 4. Although ${}^{\#}_{2}C_2^{(0)}(d_6) = 8 < {}^{\#}_{2}C_2^{(0)}(d_7) = 15$, we still have ${}_2C_2(d_6) = (1 \times 24 + 3 \times 4)/2 = 18 < {}_2C_2(d_7) = (2 \times 21)/2 = 21$. Thus, by sequentially minimizing (3.2) the MA criterion has it that d_6 is an MA design and hence better than d_7; under the effect hierarchy principle, the GMC criterion has it that d_7 is a GMC design and hence is better than d_6. In fact, although both have 9 clear main effects, d_7 has 15 clear 2fis while d_6 has only 8. Perhaps using only partial information in the AENP is a reason why sometimes the best design obtained by the MA criterion is inferior to the best one obtained by the GMC criterion under the principle above.

From (3.3), we can see that ${}_iC_j$ is a linear function of the components of ${}^{\#}_{i}C_j$ with k as the weight of ${}^{\#}_{i}C_j^{(k)}$, and sequentially maximizing the components of ${}^{\#}C$ tends to sequentially minimize the components of C. Hence, the optimal designs under the MA and GMC criteria are often consistent especially for designs with small runs (see Tables 2 and 3) but there are a significant number of cases where the two criteria yield different optimal designs. Here is one more example.

Example 2. Consider the three 2^{13-7} designs with 64 runs (designs 13-7.7, 13-7.2, and 13-7.1 in Table 4):

$$d_3: I = 12347 = 34568 = 2459 = 1456t_0 = 256t_1 = 136t_2 = 235t_3,$$
$$d_4: I = 12347 = 3458 = 2459 = 356t_0 = 256t_1 = 456t_2 = 346t_3,$$
$$d_5: I = 12347 = 34568 = 2459 = 1456t_0 = 246t_1 = 12356t_2 = 256t_3.$$

The WLPs of d_3, d_4 and d_5 are, respectively, $(0,14,28,24,24,17,12,8,0,0,0)$, $(0,26,12, 24,28,13,20,0,4,0,0)$, and $(0,14,33,16,16,33,14,0,0,0,1)$, and the most important parts of their AENPs are shown in Table 1.

Table 1. Some ${}^{\#}_{i}C_j$'s of designs d_3, d_4 and d_5.

	d_3		d_4		d_5	
${}^{\#}_{i}C_j$	$j=1$	$j=2$	$j=1$	$j=2$	$j=1$	$j=2$
$i=1$	13	13	13	13	13	13
$i=2$	78	$20,36,18,4$	78	$23,0,24,16,15$	78	$36,0,42$

According to the MA criterion, d_3 is best and d_5 is next best. However, from Table 1, one sees that they all have 13 clear main effects, d_3 has 20 clear 2fi's, d_4 has 23 clear 2fi's, and d_5 has 36 clear 2fi's. Therefore, according to the GMC and CE criteria their order of optimality should be d_5, d_4 and d_3. The best design d_3 under the MA criterion is not the best one among the three.

4. Relationship with Clear Effects Criterion

In order to study the relationship between the CE criterion and the new one, we first present two formulas for calculating the numbers of clear effects via the AENP.

Lemma 1. *Consider the 2^{n-m} designs with $R \geq III$. Then ${}^{\#}_{1}C_2^{(0)}$ is simply the number of clear main effects in a design, and ${}^{\#}_{2}C_2^{(0)} - {}^{\#}_{1}C_2^{(1)}$ is simply the number* *of clear 2fi's in a design.*

Based on Lemma 1 and the related results of Chen and Hedayat (1998), we can easily obtain the following Theorem 3, which shows that the CE criterion is the one maximizing the special functions of the AENP in Lemma l.

Theorem 3. (1) *When $n \leq 2^{n-m-1}$, the CE criterion selects the 2^{n-m} designs sequentially maximizing ${}_1^{\#}C_2^{(0)}$ and ${}_2^{\#}C_2^{(0)}$ as the optimal ones;* (2) *when $2^{n-m-1} < n < 2^{n-m} - 1$, there exist only the designs with $R \leq III$, and any 2^{n-m} design with $R = III$ has neither any clear main effect nor any clear 2fi;* (3) *for given n and m, if optimal designs under the CE criterion exist, then the GMC design must be the best one among all optimal designs under the CE criterion, where the meaning of "best" is under the comparison in Definition 1 of the GMC criterion.*

Now let us discuss the links between the MA and CE criteria. Consider the designs with $R \geq III$. From the analysis in Sections 3 and Theorem 3, we have found that the MA criterion only uses the information from $\{{}_i^{\#}C_j^{(k)}, i, j = 0, 1, \ldots, n, k = 1, \ldots, K_j\}$ at (2.1), and choosing optimal designs by the CE criterion only uses the information from $\{{}_i^{\#}C_j^{(0)}, i, j = 0, 1, \ldots, n\}$. This implies that the information used comes from the two separate parts of the set (2.1). As mentioned above, the two parts have the relation ${}_i^{\#}C_j^{(0)} + \sum_{k=1}^{K_j} {}_i^{\#}C_j^{(k)} = \binom{n}{i}$ for any i and j. Thus the larger ${}_i^{\#}C_j^{(0)}$ we choose, the smaller the number $\sum_{k=1}^{K_j} {}_i^{\#}C_j^{(k)}$ we obtain. In many cases, when ${}_i^{\#}C_j^{(0)}$ is large, the weighted sum ${}_iC_j = \sum_{k=1}^{K_j} k\, {}_i^{\#}C_j^{(k)}$ tends to be small. Thus sequentially maximizing the sequence $({}_1^{\#}C_2^{(0)}, {}_2^{\#}C_2^{(0)}, \ldots)$ tends to sequentially minimize the sequence (3.2). Perhaps this is the reason why, in many cases, the two criteria would give the same optimal designs. However, although the relationship between the number ${}_i^{\#}C_j^{(0)}$ and the sum $\sum_{k=1}^{K_j} {}_i^{\#}C_j^{(k)}$ is rather clear, the same cannot be said for ${}_i^{\#}C_j^{(0)}$ and the weighted sum ${}_iC_j = \sum_{k=1}^{K_j} k\, {}_i^{\#}C_j^{(k)}$. Therefore, conflicting results from the two criteria may appear, as shown in the examples given.

Return to consider the relationship with CE criterion. While the CE criterion cannot distinguish between designs having same numbers of clear main effects and 2fi's the new criterion can. The following example illustrates this point.

Example 3. Consider the 2^{18-12} designs with 64 runs. According to the CE criterion there are 33 best ones with 18 clear main effects and no clear 2fi's, two of them are listed in Table 4. Among the 33 designs, under the GMC criterion the best one is 18-12.1 with ${}_2^{\#}C_2 = (0, 60, 0^3, 84, 0^2, 9)$, in it there are 60 2fi's each aliased with only one 2fi. The worst one is 18-12.33 with ${}_2^{\#}C_2 = (0^3, 36, 75, 42)$, thus every 2fi of the design is aliased with at least three 2fi's, and there are seven such designs. There are 14 designs for which every 2fi is aliased with at least two 2fi's, and 2, 2, 2, 1, and 4 designs that have 32, 8, 6, 4 and 2 2fi's, respectively, each aliased with one 2fi. Obviously, for the design 18-12.1, one can easily de-alias up to 60 2fi's through the least follow-up experiments if needed. But for the other designs, one only can de-alias very few 2fi's by some follow-up experiments, or it is difficult to de-alias any 2fi's.

Accordingly, in some sense, the new criterion can be viewed as a refinement of the CE criterion. Note that CE criterion cannot be used when there are no clear effects. However there is no limitation on the use of the new criterion, and it provides more information than the CE criterion.

5. Relationship with Maximum Estimation Capacity Criterion

Cheng and Mukerjee (1998) and Cheng, Steinberg and Sun (1999) discussed the estimation capacity of a design d. Let $E_r(d)$ denote the number of models containing all the main effects and r 2fi's, $1 \leq r \leq n(n-1)/2$, which can be estimated by the design d. The design d is said to dominate a design d' if $E_r(d) \geq E_r(d')$ for all r, with strict inequality for at least one r. Furthermore, a design that maximizes $E_r(d)$ for all r is said to have maximum estimation capacity (MEC). We consider designs with $R \geq III$ in this and next sections.

Clearly, there are ${}_2^{\#}C_2^{(k)}/(k+1)$ alias sets containing $k+1$ 2fi's and ${}_1^{\#}C_2^{(k+1)}$ alias sets containing $k+1$ 2fi's and one main effect. Moreover, an alias set contains at most $l = \min\{\lfloor n/2 \rfloor, 2^m\}$ 2fi's, where $\lfloor x \rfloor$ is the integer part of x. Then all the alias sets containing 2fi's but none of the main effect can be partitioned into l classes. The i-th class consists of the alias sets that contain $i+1$ 2fi's, $i = 0, 1, \ldots, l-1$. Let $\mathcal{C}_i$ be the i-th class. Then $|\mathcal{C}_i| = {}_2^{\#}C_2^{(i)}/(i+1) - {}_1^{\#}C_2^{(i+1)}$, where $|\cdot|$ denotes the cardinality of a set. Note that $|\mathcal{C}_i|$ may be zero for some i's. From the definition of $E_r(d)$, one has the the following result.

Theorem 4. *$E_r(d)$ can be expressed as a function of ${}_2^{\#}C_2$ and ${}_1^{\#}C_2$ as*

$$E_r(d) = \begin{cases} \displaystyle\sum_{r_0+\cdots+r_{l-1}=r} \prod_{i=0}^{l-1} \binom{|\mathcal{C}_i|}{r_i} (i+1)^{r_i}, & \textit{if } r \leq f, \\ 0, & \textit{otherwise}, \end{cases} \tag{5.1}$$

where $0 \leq r_i \leq |\mathcal{C}_i|$, $f = 2^{n-m} - 1 - n$.

Thus the MEC criterion can be treated as the one that optimizes a special function of the AENP. The following discussion further illuminates this point.

Using the notation in Cheng and Mukerjee (1998), it has been shown that a design d will behave well under the MEC criterion if $\sum_{i=n+1}^{n+f} m_i(d)$ is large and $m_{n+1}(d), \ldots, m_{n+f}(d)$ are close to one another. In other words, a design d does well under MEC if $\sum_{i=n+1}^{n+f} m_i(d)$ is large and $\sum_{i=n+1}^{n+f} m_i^2(d)$ is small. Since $\sum_{i=n+1}^{n+f} m_i(d) = \sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)$ and $\sum_{i=n+1}^{n+f} m_i^2(d) = \sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)^2$, it follows that a design d that maximizes $\sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)$ and minimizes $\sum_{i=0}^{l-1} |\mathcal{C}_i|(i+1)^2$ does well under the MEC criterion.

6. Maximum Estimation Ability

The optimal designs under the MEC criterion can estimate as many models as possible that involve all the main effects and some 2fi's, under the assumption that all 2fi's not in the model and higher order interactions are negligible. However, such an assumption seems to be too strong to validate since one usually does not know whether all the 2fi's not in the model are absent. In such cases one would prefer to choose designs in which there is small degree of aliasing between the main effects and 2fi's, and between the 2fi's. We introduce the notion of estimation ability, and propose a maximum estimation ability criterion to try to avoid the above assumption.

First, under the effect hierarchy principle, the number of main effects that can be estimated in a design should be as large as possible, so we should sequentially maximize the components of $_1^\# C_2$ first to reduce the degree of the main effects aliased with 2fi's, and then sequentially maximize the components of $_2^\# C_1$ as the first step of considering 2fi's. (The $_2^\# C_1$ can be ignored when considering the designs with $R \geq III$, see Section 7.)

Next, consider the classes $\mathcal{C}_i$ for $i = 0, 1, \ldots, l-1$. Note that there are $i+1$ 2fi's in each alias set of class $\mathcal{C}_i$. Hence, a smaller i implies aliasing between the 2fi's in the alias sets of $\mathcal{C}_i$ at a lesser degree. For a given i ($i = 0, 1, \ldots, l-1$), any model, involving $s \leq n$ main effects and $r \leq \sum_{k=0}^{i} |\mathcal{C}_k|$ 2fi's in different alias sets of $\bigcup_{k=0}^{i} \mathcal{C}_k$, can be estimated under the assumption of absence of the 2fi's in the s alias sets containing the s main effects, other $k|\mathcal{C}_k|$ 2fi's in the alias sets of $\mathcal{C}_k$ for $k = 0, \ldots, i$, and the interactions involving at least three factors. For convenience, we call a model involving only the 2fi's in the alias sets of $\bigcup_{k=0}^{i} \mathcal{C}_k$ an ith-class model. A good design should sequentially maximize $|\mathcal{C}_i|$ for $i = 0, 1, \ldots, l-1$, under the condition of sequentially maximizing the components of $_1^\# C_2$ and $_2^\# C_1$. We say that such a design has maximum estimation ability (MEA). Especially, for a given i, if the design satisfies the above condition for $|\mathcal{C}_k|$ ($k = 0, 1, \ldots, i$), we say that it has MEA for the ith-class model.

Optimal designs under the MEA criterion can estimate as many models as possible that involve main effects and 2fi's with minimum confounding. If the experimenter wishes to de-alias the confounding between the effects, he/she needs only perform a few follow-up experiments.

Since $|\mathcal{C}_i| = {}_2^{\#}C_2^{(i)}/(i+1) - {}_1^{\#}C_2^{(i+1)}$, for given ${}_1^{\#}C_2$ and ${}_2^{\#}C_1$, sequentially maximizing $|\mathcal{C}_i|$ for $i = 0, \ldots, l-1$ is equivalent to sequentially maximizing the components of ${}_2^{\#}C_2$. As a result, a GMC design sequentially maximizes the estimation ability to the ith-class model for $i = 0, \ldots, l-1$, and has the MEA.

7. Simplification and More Applications of the AENP

While one may argue that the AENP of a design looks rather complicated, we emphasize that from the point of view of applications, one needs to consider only the anterior part of the AENP rather than the whole. If we consider the designs in which third and higher order interactions are negligible, we only need to consider the sub-array $({}_1^{\#}C_1, {}_0^{\#}C_2, {}_1^{\#}C_2, {}_2^{\#}C_1, {}_2^{\#}C_2)$; this can usually discriminate different designs. If we consider the designs in which only fourth and higher order interactions are negligible, the sub-array $({}_1^{\#}C_1, \ldots, {}_3^{\#}C_3)$ at (2.3) suffices. As we know, the former receives more attention in practice. From these small sub-arrays we can already obtain all the information concerning the numbers of clear main effects and 2fi's, and the severity of confounding between the lower-order effects. Then the complete AENP can be reduced to a few numbers. Especially, if we only consider the designs of the former case with $R \geq III$, we can further drop ${}_1^{\#}C_1$, ${}_0^{\#}C_2$ and ${}_2^{\#}C_1$ since ${}_2^{\#}C_1$ can be determined by ${}_1^{\#}C_2$ which precedes it, and hence need only look to $({}_1^{\#}C_2, {}_2^{\#}C_2)$ in (2.3). For the designs in Tables 2–4 in the Appendix, only these two entries are listed.

Aside from the criteria MA, CE, MEC and MEA, which can be obtained by choosing different functions of the AENP, many other criteria surface. For example, for the maximal designs of resolution IV proposed by Chen and Cheng (2006), we have that a 2^{n-m} design of resolution IV is maximal if and only if the design satisfies the two conditions: ${}_1^{\#}C_2^{(0)} = n$ and $\sum_{k\geq 1, j\geq 3} {}_j^{\#}C_2^{(k)} + \binom{n}{2} = 2^n - (n+1)2^m$.

The following is an extended example of the AENP's applications.

Example 4. Consider the 2^{9-4} designs d_6, d_7 and d_8 (they are 9-4.2, 9-4.1 and 9-4.3 in Table 3, respectively):

$$d_6 : I = 1236 = 1247 = 1258 = 13459,\ d_7 : I = 1236 = 1247 = 1348 = 23459,$$
$$d_8 : I = 1236 = 2347 = 1348 = 1249.$$

Their WLPs are $(0,0,0,6,8,0,0,1,0), (0,0,0,7,\ 7,0,0,0,1)$ and $(0,0,0,14,0,0,\ldots)$, respectively. Design d_6 has MA in all 2^{9-4} designs. Note that all three designs have ${}_1^{\#}C_2 = (9)$ and ${}_2^{\#}C_1 = (36)$, but

$${}_2^{\#}C_2(d_6) = (8,24,0,4),\ {}_2^{\#}C_2(d_7) = (15,0,21),\ {}_2^{\#}C_2(d_8) = (8,0,0,28).$$

So, from their AENPs, it is easily seen that all 9 main effects are clear, d_6 and d_8 only have 8 clear 2fi's and d_7 has 15. According to the GMC and CE criteria, d_7 is best.

With the AENP, one can sometimes see that some clear 2fi's are strongly clear. Consider the designs d_6 and d_8 again. The both have eight clear 2fi's. Note that ${}^{\#}_{2}C_3^{(0)}(d_8) = (36)$, which means that none of the 2fi's of d_8 is aliased with any three-factor interaction (3fi), and hence the eight clear 2fi's of d_8 are all strongly clear. For design d_6, starting from ${}^{\#}_{2}C_2(d_6)$, by carefully analyzing ${}^{\#}_{2}C_2(d_6) = (8, 24, 0, 4)$, ${}^{\#}_{2}C_3(d_6) = (4, 0, 24, 0, 8)$, ${}^{\#}_{3}C_2(d_6) = (28, 32, 24)$, ${}^{\#}_{3}C_3(d_6) = (0, 24, 24, 36)$ and ${}^{\#}_{3}C_1(d_6) = (60, 24)$, we conclude that all its aliasing cosets involving 2fi's and 3fi's, have only the following five forms: one coset containing four 2fi's but no 3fi's, one coset containing four 3fi's but no 2fi's, twelve cosets containing two 2fi's and two 3fi's, eight cosets containing one 2fi and four 3fi's, and eight cosets containing three 3fi's but no 2fi's. Then for the eight clear 2fi's of d_6 shown in ${}^{\#}_{2}C_2(d_6)$, each must be in one of the 8 cosets containing one 2fi and four 3fi's. Since every one of such 8 cosets contains 3fi's, it follows that none of the eight clear 2fi's of d_6 is strongly clear. This analysis cannot be done using WLPs alone.

8. GMC Designs of 16, 32 and 64 runs

We have obtained all GMC designs of 16 and 32 runs, and the GMC designs, up to 26 additional columns, of 64 runs. These are listed, respectively, in Tables 2, 3 and 4, and including some simple comparisons with the results of the MA and CE criteria. Some explanations are given below.

Let $a_1, \ldots, a_5$ and a_6 denote the six independent columns $(000001)'$, $(000010)'$, $(000100)'$, $(001000)'$, $(010000)'$ and $(100000)'$, respectively. Then any product of a_1, a_2, a_3, a_4, a_5, and a_6 also corresponds to a binary sequence, for example $a_1a_3a_5$ corresponds to $(010101)'$. These binary sequences are converted into decimal ones as usual, for example, 28 corresponds to $(011100)'$. We omit the converted table to save space. A 2^{n-m} design can be obtained by selecting a subset of n columns of $C = \{1, \ldots, 2^{n-m} - 1\}$, consisting of $n - m$ independent columns and m additional columns.

For simplicity, in the tables we use n-m.i to denote the i-th good design, according to the GMC criterion, among 2^{n-m} designs with n factors and m independent defining words. The additional columns are listed in decimal in the second part of the tables. The third part is the AENP of the design, here we only list ${}^{\#}_{1}C_2$ and ${}^{\#}_{2}C_2$. We also list the WLP (A_3 to A_6) and the numbers $\{c_1, c_2\}$ of clear main effects and clear 2fi's for comparison in the fourth and fifth parts, respectively. In the last part, the optimality order-numbers of the designs under the GMC, MA and CE criteria, respectively, in all the non-isomorphic

2^{n-m} designs are listed, where the subscript s of r_s in this part indicates the number of non-isomorphic designs which have the same order-number r under its corresponding criterion, and "-" means no clear main effects and 2fi's. To save space, we only list a few designs under the GMC criterion, these are best, or are used as examples in the paper. For parameters $n = 2^{n-m} - i$, $i = 1, 2, 3$, the design is unique up to isomorphism and hence is omitted. Also the best design with one additional column is omitted.

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Appendix

Table 2. 16-run GMC designs and comparisons with the MA and CE criteria.

Design	Additional Columns	AENP	WLP	Cs	Order
		$^{\#}_{1}C_2; ^{\#}_{2}C_2$	$A_3, \ldots, A_6$	c_1,c_2	G,M,C
6-2.1	14 7	6; 0,12,3	0,3,0,0	6,0	1,1,1
6-2.3	12 6	1,4,1; 9,6	2,1,0,0	1,5	3,4,3
6-2.4	12 3	0,6; 15	2,0,0,1	0,9	4,3,4
7-3.1	14 7 11	7; 0^2,21	0,7,0,0	7,0	1,1,1
7-3.3	12 6 10	1,0,6; 6,12,3	4,3,0,0	1,6	3,5,3
7-3.5	12 6 3	0,5,2; 9,12	3,2,1,1	0,4	5,3,4
8-4.1	14 7 11 13	8; 0^3,28	0,14,0,0	8,0	1,1,1
8-4.2	14 7 3 5	2,0,6; 0,24,0,4	4,6,4,0	2,0	2,4,2
8-4.3	14 7 11 3	1,6,0,1; 7,0,21	3,7,4,0	1,1	3,2,4
8-4.4	12 6 10 14	1,0^2,7; 7,0,21	7,7,0,0	1,7	4,6,3
8-4.5	14 7 3 12	0,4,4; 4,18,6	4,5,4,2	0,0	5,3,-
9-5.1	14 7 11 13 3	0,8,0^2,1; 8,0^2,28	4,14,8,0	0,0	1,1,-
9-5.2	14 7 11 3 6	0,2,5,2; 2,12,18,4	6,10,8,4	0,0	2,3,-
9-5.3	12 6 10 14 3	0,2,0,6,1; 2,12,18,4	8,10,4,4	0,0	3,5,-
9-5.4	14 7 3 12 9	0^2,9; 0,18,18	6,9,9,6	0,0	4,2,-
9-5.5	14 7 3 12 6	0^2,6,3; 0,18,18	7,9,6,6	0,0	5,4,-
10-6.1	14 7 11 13 3 6	0^2,8,0,2; 0,16,0,24,5	8,18,16,8	0,0	1,1,-
10-6.3	12 6 10 14 3 5	0^2,3,4,3; 0,6,27,12	10,16,12,12	0,0	3,4,-
10-6.4	14 7 3 12 6 15	0^3,10; 0^2,45	10,15,12,15	0,0	4,3,-
11-7.1	14 7 11 13 3 6 12	0^3,8,3; 0^2,24,16,15	12,26,28,24	0,0	1,1,-
11-7.2	14 7 11 13 3 6 5	0^3,8,0,3; 0^2,24,16,15	13,26,24,24	0,0	2,3,-
11-7.3	14 7 3 12 9 6 5	0^3,5,6; 0^2,15,40	13,25,25,27	0,0	3,2,-
12-8.1	14 7 11 13 3 6 12 9	0^4,12; 0^3,48,0,18	16,39,48,48	0,0	1,1,-

Table 3. 32-run GMC designs and comparisons with the MA and CE criteria.

Design	Additional Columns	AENP	WLP	Cs	Order
		${}^{\#}_{1}C_2; {}^{\#}_{2}C_2$	$A_3,\ldots,A_6$	c_1,c_2	G,M,C
7-2.1	30 7	7; 15,6	0,1,2,0	7,15	1,1,1
8-3.1	30 7 11	8; 13,12,3	0,3,4,0	8,13	1,1,1
9-4.1	30 7 11 13	9; 15,0,21	0,7,7,0	9,15	1,2,1
9-4.2	30 7 11 19	9; 8,24,0,4	0,6,8,0	9,8	$2,1,2_2$
9-4.3	28 14 22 26	9; $8,0^2,28$	0,14,0,0	9,8	$3,5,2_2$
10-5.1	30 7 11 19 29	10; $0,40,0^2,5$	0,10,16,0	10,0	$1,1,1_4$
11-6.1	28 14 22 26 7 11	11; $0^2,24,16,15$	0,26,0,24	11,0	$1,2,1_2$
11-6.2	28 14 7 19 25 11	11; $0^2,15,40$	0,25,0,27	11,0	$2,1,1_2$
12-7.1	28 14 22 26 7 11 13	12; $0^3,48,0,18$	0,39,0,48	12,0	$1,2,1_2$
12-7.2	28 14 7 19 25 11 13	12; $0^3,36,30$	0,38,0,52	12,0	$2,1,1_2$
13-8.1	28 14 22 26 7 11 13 19	13; $0^4,60,18$	0,55,0,96	13,0	1,1,1
14-9.1	28 14 22 26 7 11 13 19 21	14; $0^5,84,7$	0,77,0,168	14,0	1,1,1
15-10.1	28 14 22 26 7 11 13 19 21 25	15; $0^6,105$	0,105,0,280	15,0	1,1,1
16-11.1	28 14 22 26 7 11 13 19 21 25 31	16; $0^7,120$	0,140,0,448	16,0	1,1,1
17-12.1	28 14 22 26 7 11 13 19 21 25 31 3	$0,16,0^6,1$; $16,0^6,120$	8,140,112,448	0,0	1,1,-
18-13.1	28 14 22 26 7 11 13 19 21 25 31 3 6	$0^2,16,0^5,2$; $0,32,0^5,112,9$	16,148,224,560	0,0	1,1,-
19-14.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12	$0^3,16,0^4,3$; $0^2,48,0^4,96,27$	24,164,344,784	0,0	1,1,-
20-15.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9	$0^4,16,0^3,4$; $0^3,64,0^3,96,0,30$	32,189,480,1120	0,0	1,2,-
20-15.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24	$0^4,16,0^3,4$; $0^3,64,0^3,72,54$	32,188,480,1128	0,0	2,1,-
21-16.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24	$0^5,16,0^2,5$; $0^4,80,0^2,64,36,30$	40,221,640,1600	0,0	1,2,-
21-16.2	28 14 22 26 7 11 13 19 21 25 31 3 6 12 24 17	$0^5,16,0^2,5$; $0^4,80,0^2,40,90$	40,220,641,1608	0,0	2,1,-
22-17.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18	$0^6,16,0,6$; $0^5,96,0,64,0,60,11$	48,263,832,2224	0,0	1,1,-
23-18.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18 23	$0^7,16,7$; $0^6,112,64,0^2,77$	56,315,1064,3024	0,0	1,1,-
24-19.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18 23 29	$0^8,24$; $0^7,192,0^3,84$	64,378,1344,4032	0,0	1,1,
25-20.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18 23 29 5	$0^9,24,0^2,1$; $0^8,216,0^2,84$	76,442,1656,5376	0,0	1,1,-
26-21.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18 23 29 5 10	$0^{10},24,0,2$; $0^9,240,0,72,13$	88,518,2032,7032	0,0	1,1,-
27-22.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18 23 29 5 10 20	$0^{11},24,3$; $0^{10},264,48,39$	100,606,2484,9064	0,0	1,1,-
28-23.1	28 14 22 26 7 11 13 19 21 25 31 3 6 12 9 24 18 23 29 5 10 20 27	$0^{12},28$; $0^{11},336,0,42$	112,707,3024,11536	0,0	1,1,-

Table 4. 64-run GMC designs and comparisons with the MA and CE criteria.

Design	Additional Columns	AENP $^{\#}_{1}C_2;^{\#}_{2}C_2$	WLP $A_3,\ldots,A_6$	Cs c_1,c_2	Order G,M,C
8-2.1	60 15	8; 28	0,0,2,1	8,28	1,1,1
9-3.1	60 15 22	9; 30,6	0,1,4,2	9,30	1,1,1
10-4.1	60 15 22 39	10; 33,12	0,2,8,4	10,33	1,1,1
11-5.1	60 15 22 39 21	11; 34,18,3	0,4,14,8	11,34	1,1,1
12-6.1	60 15 22 39 21 59	12; 36,24,6	0,6,24,16	12,36	1,1,1
13-7.1	60 15 22 39 21 59 19	13; 36,0,42	0,14,33,16	13,36	1,2,1
13-7.2	60 14 22 11 19 7 13	13; 23,0,24,16,15	0,26,12,24	13,23	2,37,2
13-7.7	60 15 22 39 19 41 26	13; 20,36,18,4	0,14,28,24	13,20	7,1,7
14-8.1	60 14 22 11 19 7 13 21	14; 25,0^2,48,0,18	0,39,16,48	14,25	1,42,1_2
14-8.17	60 15 22 35 26 37 19 46	14; 8,52,18,8,5	0,22,40,36	14,8	17,1,17
15-9.1	60 14 22 11 19 7 13 21 26	15; 27,0^3,60,18	0,55,22,96	15,27	1,40,1
15-9.27	60 15 22 35 26 37 19 46 59	15; 0,60,30,0,15	0,30,60,60	15,0	27,1,27
16-10.1	60 14 22 11 19 7 13 21 26 25	16; 29,0^4,84,7	0,77,28,168	16,29	1,45,1
16-10.20	60 15 22 35 26 37 19 46 59 29	16; 0,36,66,0^2,18	0,43,81,96	16,0	20,1,19_{30}
17-11.1	60 14 22 11 19 7 13 21 26 25 31	17; 31,0^5,105	0,105,35,280	17,31	1,38,1
17-11.17	60 15 22 35 26 37 19 49 29 55 41	17; 0,18,81,16,0^2,21	0,59,108,150	17,0	17,1,17_{24}
18-12.1	60 14 22 11 19 7 13 21 38 25 31 58	18; 0,60,0^3,84,0^2,9	0,92,112,280,	18,0	1,3,1_{33}
18-12.6	60 15 22 35 26 37 19 49 29 55 41 50	18; 0,6,75,48,0^3,24	0,78,144,228,	18,0	6,1,1_{33}
19-13.1	60 15 22 35 26 49 37 55 19 50 29 46 41	19; 0^2,48,96,0^4,27	0,100,192,336	19,0	1,1,1_{25}
20-14.1	60 15 22 35 26 49 37 55 19 50 29 46 41 59	20; 0^3,160,0^5,30	0,125,256,480	20,0	1,1,1_{24}
21-15.1	56 28 44 52 14 22 26 38 42 50 62 7 11 19 13	21; 0^4,80,0^2,64,36,30	0,221,0,1600	21,0	1,16,1_{16}
21-15.16	56 11 22 37 7 59 28 42 14 49 19 38 21 41 26	21; 0^5,60,126,24	0,204,0,1680	21,0	16,1,1_{16}
22-16.1	56 28 44 52 14 22 26 38 42 50 62 7 11 19 13 21	22; 0^5,96,0,64,0,60,11	0,263,0,2224	22,0	1,15,1_{15}
22-16.14	56 11 22 37 7 59 28 42 14 49 19 38 21 41 26 44	22; 0^5,6,105,120	0,250,0,2304	22,0	14,1,1_{15}
23-17.1	56 28 44 52 14 22 26 38 42 50 62 7 11 19 13 21 25	23; 0^6,112,64,0^2,77	0,315,0,3024	23,0	1,9,1_9
23-17.8	56 11 22 37 7 59 28 42 14 49 19 38 21 41 26 44 13	23; 0^6,28,144,81	0,304,0,3105	23,0	8,1_2,1_9
23-17.9	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35	23; 0^6,21,168,54,10	0,304,0,3105	23,0	9,1_2,1_9
24-18.1	56 28 44 52 14 22 26 38 42 50 62 7 11 19 13 21 25 31	24; 0^7,192,0^3,84	0,378,0,4032	24,0	1,8,1_8
24-18.8	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38	24; 0^7,48,198,30	0,365,0,4138	24,0	8,1,1_8
25-19.1	56 28 44 52 14 22 26 38 42 50 62 7 11 19 13 21 25 31 35	25; 0^8,216,0^2,84	0,442,0,5376	25,0	1,5,1_5
25-19.5	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52	25; 0^8,90,210	0,435,0,5440	25,0	5,1,1_5
26-20.1	56 28 44 52 14 22 26 38 42 50 62 7 11 19 13 21 25 31 35 37	26; 0^9,240,0,72,13	0,518,0,7032	26,0	1,4,1_4
26-20.4	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 25	26; 0^9,160,165	0,515,0,7062	26,0	4,1,1_4
27-21.1	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 25 31 44	27; 0^{10},264,48,39	0,606,0,9064	27,0	1,2,1_2
27-21.2	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 55 25	27; 0^{10},231,120	0,605,0,9075	27,0	2,1,1_2
28-22.1	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 25 31 44 41	28; 0^{11},336,0,42	0,707,0,11536	28,0	1,2,1_2
28-22.2	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 55 25 31	28; 0^{11},300,78	0,706,0,11548	28,0	2,1,1_2
29-23.1	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 55 25 31 44	29; 0^{12},364,42	0,819,0,14560	29,0	1,1,1
30-24.1	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 55 25 31 44 41	30; 0^{13},420,15	0,945,0,18200	30,0	1,1,1
31-25.1	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 55 25 31 44 41 62	31; 0^{14},465	0,1085,0,22568	31,0	1,1,1
32-26.1	56 11 22 37 7 59 28 42 14 49 13 26 47 50 19 21 35 38 52 55 25 31 44 41 62 61	32; 0^{15},496	0,1240,0,27776	32,0	1,1,1

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